A STUDY OF ICE FORMATIONS AND THEIR EFFECTS ON BRIDGES

A HIGHWAY RESEARCH REPORT

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A STUDY OF ICE FORMATIONS AND THEIR EFFECT ON BRIDGES

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INTRODUCTION

In the northern latitudes of the United States, large quantities of ice are manufactured and stored on the rivers and streams during the winter months. On some rivers this build-up of ice may result in flooding of the river causing entire flood plains to be covered with ice and resulting in possible damage to facilities located in the area. Other rivers may simply form ice covers along the side or over the entire river with no overflow of the river banks, and some rivers or parts of the river will store no ice at all.

In the spring when the break-up of the vast quantities of stored ice occurs, it is carried down stream by the water as large floes and natural or man made obstacles may cause them to jam, resulting in damming of the river. The jamming of these ice floes oftentimes causes flooding and considerable damage to the surrounding areas. If the obstacle causing the jam is a man made object, such as a bridge, the forces exerted against the structure by the ice may cause considerable damage to, or even complete destruction of it.

Such cases of damage to bridges have occurred in the past few years here in Montana. A fairly new bridge built in the latter 1950's on the State Primary System at Glendive, Montana

ended up with a badly cracked pier after the spring ice run of 1962. A picture showing the crack is given as figure 1. Calculations were made to determine the approximate magnitude of load necessary to cause the damage indicated. Since the actual distribution of loading was unknown, both a concentrated load acting at the level of the crack (assumed at 17 ft. below the top) and a uniform load over a portion of the pier depth were assumed. The approximate value calculated for the concentrated load was 175^K. The magnitude of the total uniform load, calculated assuming that it was distributed from a distance of $7\frac{1}{4}$ ft. down from the top of the pier to the top of the footing, was approximately 380^K. It is expected that the actual loading would have been considerably more complex than those assumed, but that the total magnitude of load would have been somewhere between these two values given. Calculations of this nature give some indication of the large lateral loads that can be expected.

A truss bridge across the West Gallatin river at Logan, Montana was pushed off its abutments and completely destroyed by the force of an ice jam behind it in the spring of 1963. A picture of the ice jammed behind the bridge is given as figure 2.

Another truss bridge across the Madison river 18 miles above Ennis, Montana was threatened when the ice built up under it during the period of ice cover formation in the winter of 1962-63.

Figure 1. Cracked Pier - Glendive, Montana, 1962

Figure 2. Ice Shoving on Bridge. Logan, Montana, 1963

Figure 3. Ice Jam on Missouri River Near Townsend

Figure 4. Ice Jam on Missouri River Near Townsend

Figure 5. Ice Jam on Missouri River Near Townsend

It was necessary to close the bridge to traffic for a short time and reports from highway personnel in the area indicated that rather large, lateral, elastic deformations were noted. No major damage occurred, however, and the bridge is still open to traffic.

There are also numerous reports[1, 2, 3, 4, 5, 6, 7] in the literature indicating damage to bridges, dams, and highways resulting from the forces of ice. Damage from both the thermal expansion of the ice and the jamming of the ice floes are noted.

Even more prevalent is the damage resulting from the flooding of the river as a result of the ice jamming. This flooding can wash out roadfill and approaches, weaken the abutments and piers, or wash mud and debris across the highways. All of which, although not major, does result in a considerable cost in yearly maintenance.

The present criteria for the design of bridge piers to resist ice forces as specified by The American Association of State Highway Officials aregiven as follows:

> "Pressure of ice on piers shall be calculated at 400 pounds per square inch. The thickness of ice and height at which it applies shall be determined by investigation at the site of the structure."[8]

Such a requirement does nothing more than to make the engineer aware of the fact that there might be an ice load which should be included in the design. It does not even indicate whether or not

the pressure should be applied in both directions on the pier. The best that a bridge engineer can do, using such criteria and other presently available knowledge, is to make an educated guess based on past experiences.

With the increasing costs of bridge construction and the increase in the number of bridges being built, it seemed that a better understanding of the nature of ice formation and the forces exerted on bridges by the ice was necessary. Therefore, this project was undertaken as the first step toward getting this better understanding of the problems involved. The project is an investigation of the formation of ice on the rivers, the effects which bridges have on this formation and the forces which can be expected to be exerted on the bridges by the ice. It is the objective of this present project to determine what work has been done in this field and in what direction a more intensive research effort should go in order to be able to predict the formation of ice and the forces exerted on bridges by this ice, thus eventually establishing reliable design criteria.

The ice loads to which a bridge pier may be subjected can come from three sources. These are the thermal expansive force of a solid ice sheet, the dynamic force of a moving ice floe, and the static forces of an ice jam formed against or through the bridge piers. Not all bridge sites will be conducive to the

conditions necessary for the three sources of loading to occur nor is it possible, at such sites where they all could occur, that they would act simultaneously.

The maximum force resulting from the thermal expansion of an ice sheet would occur during a period of rapid temperature rise prior to the spring ice break-up and run. Ice jams can occur and cause a force on a bridge or pier either during the period of ice cover formation or spring ice run. Ice floes of significant size would only occur during the spring ice run. However, the maximum forces expected from jams and floes could not act simultaneously since no ice would be flowing past the bridge when a jam was formed there.

A separate coverage of each source of loading is given. No attempt has been made in these discussions to establish any design criteria. A report on the work which is available in the literature as applied to the problem of forces against bridges is given. Comparison between various approaches to the problem are made and attempts are made to explain the reasons for any variations in results. Areas where it appears that additional work is necessary are pointed out. Where possible, sample calculations are given for an assumed bridge and site to give some idea to the reader of the possible magnitudes of pressures which may occur from the various types of loading.

It is hoped that, with many additional investigations, it will be possible to eventually predict the maximum load from any

and all sources of loading for a particular river and site. From such knowledge it will then be possible to establish reasonable design criteria.

Also, it was proposed to make a study of instrumentation for the measurement of forces exerted on the bridge piers by the ice. A preliminary study into the problem was made along with the detailed study of the ice forces that can be expected to act against the piers. The complexity of the overall problem and the numerous ways in which ice can possibly exert a force on a bridge, requires that a separate study be made at each particular site chosen for study. Different types of instrumentation will be required to measure the forces from the various sources of ice loading, and so it is necessary to know which sources of ice loading are possible at the particular site chosen and then decide which measurements are desired. Thus, from these studies, it was concluded that, until a definite plan for the continuation of this work is decided upon or a definite site is chosen for instrumentation and study, little value would come from a further study or development of such instrumentation.

The following chapters in Part I present a discussion of the ice loads that may be exerted against a bridge. The physical properties of ice and a discussion of the phenomenon of ice cover formations are included in Part II.

PART I

CHAPTER 1

EXPANSIVE FORCES OF ICE AGAINST BRIDGE PIERS

The pressure resulting from expanding ice can be a very important factor in determining the cross-section of a dam or bridge pier.

The expansion of ice which is of concern, is not the expansion that occurs during the freezing process which is most familiar, but the thermal expansion caused by a rapid increase in temperature following a sustained cold spell. This usually occurs during the late winter and early spring. All materials when subjected to a change in temperature will also change in volume, the amount of change is dependent upon the type of material and is measured by the coefficient of thermal expansion.

In order for there to be a force created resulting from the thermal expansion, the ice must be confined and free of cracks or other discontinuities that could relieve the force. Consequently, only clear sheet ice is capable of developing appreciable thermal stresses.

The formation of a clear sheet of ice requires a condition of stationary or slowly moving water. Most rivers in Montana have average winter flow velocities greater than that which will permit the formation of a smooth ice sheet. However, the velocity is not

uniform throughout the river and along the edges it may be slow enough to permit an ice sheet to form. If the water velocity between the end abutment and first pier, or any other span, is sufficiently slow to permit an ice sheet formation while the adjacent spans remain clear, a situation is developed whereby thermal expansion of the ice would result in a force against the pier.

Sheet ice will begin to form in placid water whenever the water reaches a temperature slightly below 32°F. Long needle-like crystals of ice form over the surface of the water starting from the edges and progressing toward the center. These crystals join together until eventually the entire surface becomes covered with ice and solidifies into a thin sheet. Thickening of the ice takes place downward by conduction. This requires that the air temperature remain considerably below freezing and, as the ice thickens, the top surface will continue to cool. Cooling of the top surface causes it to contract and, since the tensile strength of ice is quite low (Approx. 100 #/in.^{2})[14] cracking will occur. Generally these cracks will fill with water and freeze leaving a solid surface. The ice sheet will continue to thicken as long as there is sufficient differential in temperature between the bottom of the ice at 32°F and the air at the top. Thicknesses up to 24 inches or better would not be uncommon for Montana. Any increase in

temperature will cause the ice to expand and, with restraint, compressive forces will develop. In the cases when the cracks caused by contraction of the outer surface upon cooling do not fill with water and freeze solid or if there are other discontinuities in the ice sheet, the forces of expansion will be much less. If the cracks caused by the contraction of the ice are very small, the water may not be able to fill them and freeze. Such a case could occur in nature if the area of the sheet is very small as might possibly be the situation with the space between bridge piers. A considerably smaller thrust would be expected for such a case.

Early concern of the ice expanding against water structures, particularly dams, initiated the use of arbitrarily chosen design loads.[32] One such value used and still cited in a more recent source[33] is 50 k/ft. This value can be approximated by using a limiting value for the crushing strength of ice of 400 $\#/in.^2$ and assuming that the stress varies linearly from 400 $\#/in^2$ at the top to 0 $\#/in.^2$ at the water surface of a 20 in. thick ice sheet. For this case, $F = \frac{(12)\ (400)\ (20)}{(2)\ (1000)} = 48$ K/ft. which when rounded off would equal the 50 K/ft. This was felt to be a limiting value for the expansive force assuming that the temperature gradient as shown in figure 1-1 is linear, varying from the air temperature considerably below freezing at the top, to $32^{\circ}F$ at the bottom, or

water surface. However, such a design load was felt by Brown and Clarke[32] to be too severe, particularly when they considered the large number of low dams which are still standing and are quite vulnerable to a load of such a magnitude but were not designed for any ice expansive load at all. This prompted some research into the problem in an attempt to ascertain realistic values for ice pressure to be used in dam design.[32, 34, 35] The research done verified that the expansion forces which can be expected are much lower than indicated above and at the same time it pointed out the complexity of the problem.

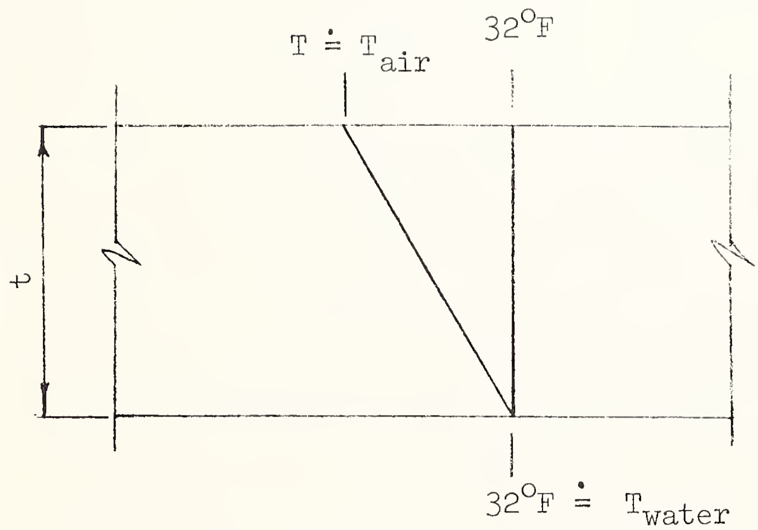


Figure 1-1

Temperature Gradient Through The Ice Sheet

First it should be noted that the usual equation used to calculate the expansive force of an elastic member, equation 1-1, is not valid even if the variation in temperature gradient is

taken into account. This is because ice is not an elastic solid. It behaves as a viscous solid, flowing under applied load. The extent of flow varies with the time, temperature, and load. Also, according to Dorsey,[14] the orientation of the ice crystals will affect the rate of yielding. The ice crystals in sheet ice are orientated with the optical axis normal to the freezing surface which allows less yielding than if the optic axis was parallel with the stress. However, Strong[15] pointed out that this is perhaps not true in that tests which he ran varied only 1.5% between the strengths parallel and perpendicular to the optic axis.

$$F = \alpha \Delta T A E \qquad (1\text{-}1)$$

where:

F is the total thermal force in #.

α is the coefficient of expansion in in./in./°F.

(for ice $\alpha \times 10^6 = 52.52 - 0.1852t + 0.00885t^2 - 0.00237t^3$ in./in./°C where t is the temperature in °C. To use equation 1, it is necessary to convert α into the units of in./in./°F.)

ΔT is the temperature change in °F.

A is the cross-sectional area in in.^{2}

E is the modulus of elasticity for the material in #/in.^{2}

Another factor which was overlooked in the original estimates of expansive loads was the buckling of the ice sheet. The buckling load for ice is a function of the geometric and elasto-plastic properties of the ice sheet. Rose[34] gives a value of 35.4 k/ft. as the theoretical buckling strength of a 165 ft. square ice sheet

20 inches thick. It is further noted that buckling generally does not occur, as indicated by pressure ridges, in ice sheets thicker than 12 inches. However, buckling has been known to occur in ice sheets as thick as 20 inches but with dimensions larger than given above to calculate the 35.4 k/ft. load. This was also verified by Lofquist[35] although the value which he calculated for the buckling load of a 20 in. thick cover was only 23.6 k/ft. which is considerably lower than that given by Rose. The reason for this is that any theory developed for calculating the buckling load is based on a perfect elastic material. Ice is not a perfect elastic material as was mentioned previously, and, as such, requires a choice of a modulus of elasticity which, at the present time, can only be a guess and consequently results in a large range of possible values for the buckling load. However, there is an indication that the maximum load to expect could be considerably less than the larger value mentioned.

Measurements made on numerous dams in the Colorado Rockies[35] also indicated lower forces could be expected. These measurements were made with the use of both electrical and indentor type gages which were placed at the top, middle, and bottom of the ice sheet. From the electrical gages, a fairly good picture of the stress distribution could be determined. This was not always a linear stress distribution from top to bottom which was exactly as expected. The indentor gages only recorded the maximum stress

during the year and since it is not expected that the maximum stress at all points occurs at the same time, the value of thrusts calculated from the indentor gage data could be in some error.

The maximum loads calculated from the measurements varied from year to year and from one reservoir to another. The highest maximum calculated was for Eleven Mile Canyon Reservoir which was 20 k/ft in the winter of 1949-50. While at Antero Reservoir the maximum recorded in the winter of 1950-51 was only 3.6 k/ft. The same year the maximum thrust recorded at Tarryall Reservoir was 17 k/ft.

Part of the reason given for this large variation in maximum thrust was attributed to the shores. If the shores yield under thrust, it can be expected that the maximum load would be reduced. Thus, for steep rocky shores, the maximum load would be expected to be greater than for flat shores. Exception to this generality would occur if there was considerable bond between the ice and a flat shore. Little or no relief could be expected for ice confined between two bridge piers so that the maximum thrust could be expected.

Other measurements that were taken were the air temperature, ice temperature, rate of temperature rise, and thickness of ice. Unfortunately, these measurements were not recorded as being simultaneous with the stress measurement and because of this no correlation can be made between the various measurements taken.

The maximum extreme measurements recorded for the air temperature was a differential of $116^{\circ}F$ from $-50^{\circ}F$ to $+66^{\circ}F$ in the two months of January and February. The maximum rate of ice temperature rise recorded was $7^{\circ}F$/hr. and the depth of ice at Eleven Mile Canyon Reservoir varied between 20 and 24 inches.

Other factors which tended to have a considerable effect on the absorption of heat in an ice sheet were the snow cover and solar radiation. Snow on the ice insulates it from the atmosphere and the temperature change is consequently much less. Solar radiation on the other hand increases the rate of temperature rise. Rose[34] calculated the increase in thrust when solar radiation was present and found it amounted to 20% over the case when it was neglected.

Daily temperature variation of 20 to $30^{\circ}F$ and even larger weekly variations can be expected. Accompanying the temperature variations are also daily variations in the thrust. The magnitude of thrust, however, is not as dependent upon the total magnitude of the temperature variation as it is dependent on the rate of ice temperature rise. If the rate of temperature rise is very slow, the ice, being a viscous solid, has time to flow and relieve itself of the pressure before high thrusts can be developed. A high rate of ice temperature change is therefore necessary if high thrusts are going to be developed before flow can occur to reduce it. This requires that there must also be a rapid rise in temperature. Such

rapid air temperature changes are quite common in the chinook wind belts and rises in temperature of 5 to 15°/hr. would not be at all uncommon.

One case of very extreme temperature variations occurred in the Black Hills of South Dakota in January of 1943.[36] Here the air temperature fluctuated with rises and drops of 50 to 60°F in a matter of minutes. Although local chinook effects could have possibly contributed to these conditions it was felt that the phenomenon was essentially the wavering motion of a pronounced quasi-stationary front separating Maritime Polar air from Continental Arctic air.

Because ice was found not to be a perfect elastic material, the classical methods used to calculate the thermal stresses are not applicable and new methods have to be developed. This resulted in some research being initiated in an effort to obtain new methods to use in the predicting of ice thrust resulting from thermal expansion.

The first work recorded was by Brown and Clarke[32] who ran laboratory tests to determine the rate of pressure increase as a function of the rate of temperature increase. The results of this work are plotted on figures 1-2 and 1-3. Unfortunately, only two tests were run from which the data was obtained to plot the curve which, for experimental work, would generally not be sufficient.

RELATIONSHIP BETWEEN TEMPERATURE AND PRESSURE RISE PER HOUR

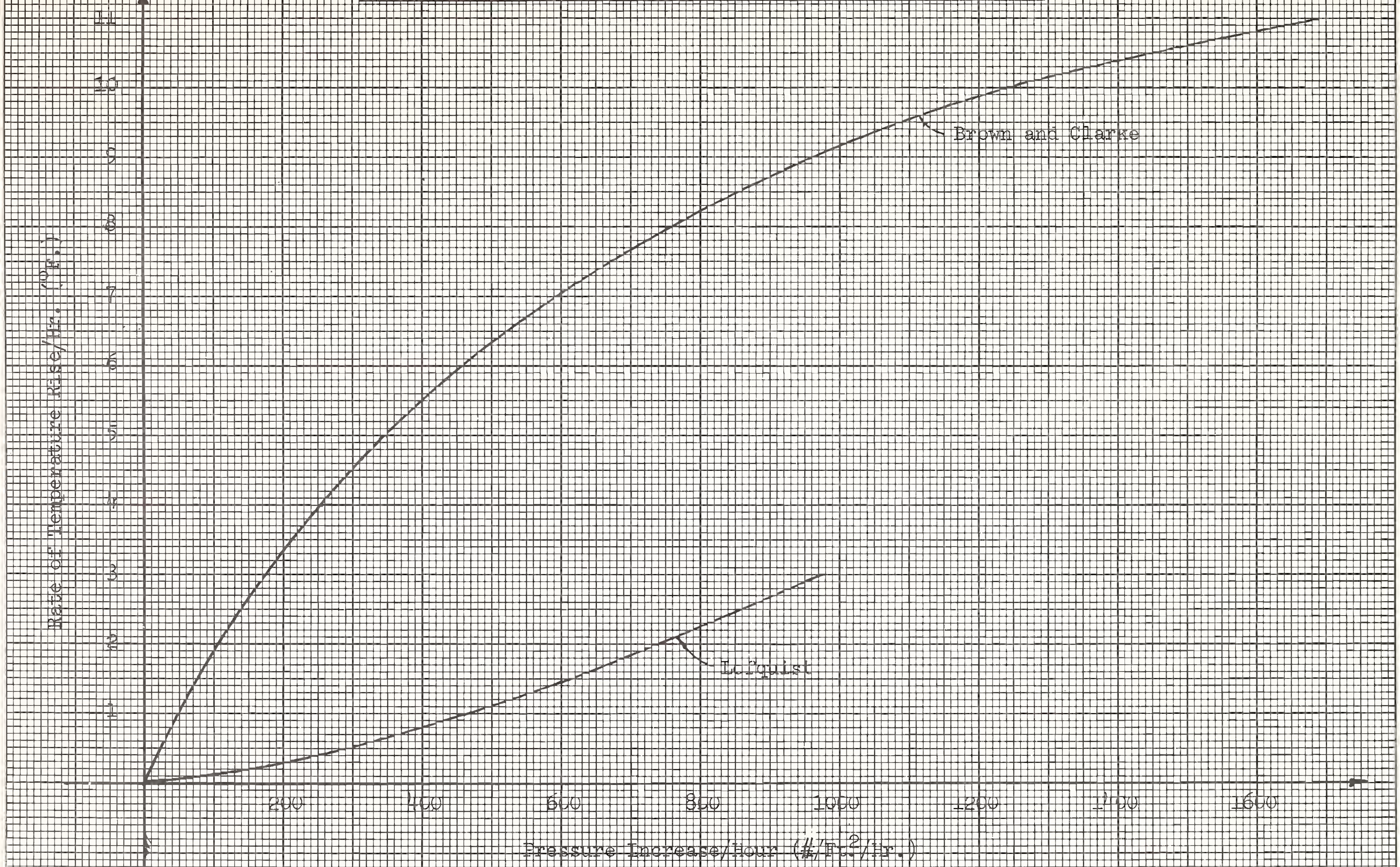


FIGURE 1-2

RELATIONSHIP BETWEEN TEMPERATURE AND PRESSURE RISE PER HOUR

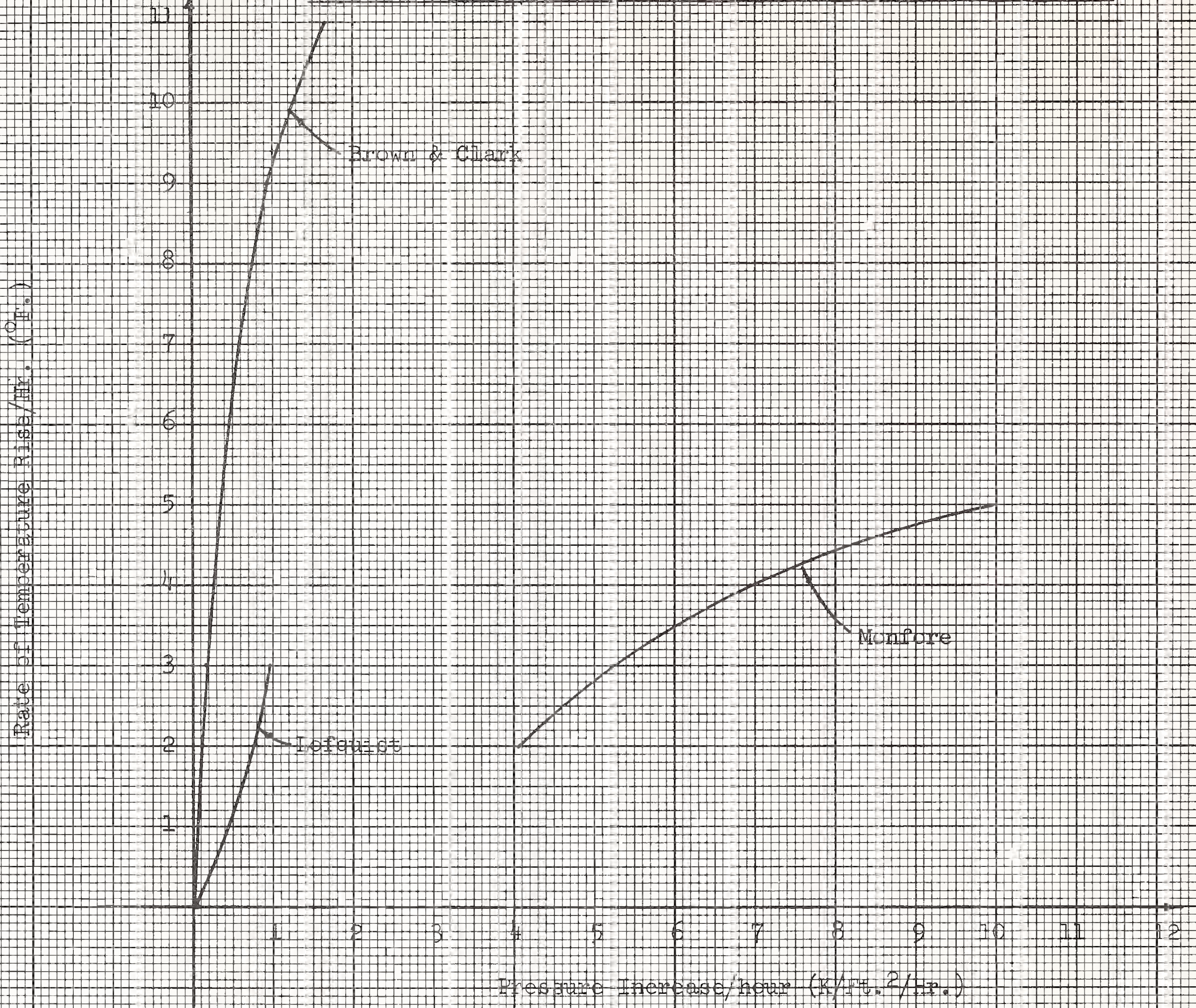


FIGURE 1-3

Three inch cubes were used for these tests. These were placed in a small testing machine and were completely insulated from the outside air by a box. Space was left between the box and the cube so that air could be easily circulated. Once the temperature of the ice and surrounding air was lowered to the desired temperature by dry ice, this dry ice was so manipulated as to permit a uniform increase in air temperature. The force caused by the thermal stresses could be obtained by reading the scale of the testing machine.

It is noted that a couple of things were not accounted for in these tests. First, interpretation of the results of this work would indicate that it was assumed there was no variation in the temperature from one point in the cross section to the next. Actually, however, the diffusivity of the temperature would not be expected to be uniform. Instead the rate of temperature change of the exterior would be expected to be considerably greater than the rate of change at the center.

Also, no lateral restraint was given the test sample as would generally be the case of ice in the natural state. This would tend to permit increased lateral flow and a reduced thermal stress.

Because of the lack of more test data, the lack of lateral restraint, and the temperature variation that would be present across the cross-section, it is felt that considerable error could be present in the data so given. It does, however, give

some idea as to behavior of the ice when subjected to temperature increases.

The tests demonstrated the flow, or yielding, of ice at different rates of loading, and under sustained loads of different intensities. At a given temperature and a given increment of load, the amount of yielding increases with a lengthening of the time interval between increments of load. Under a sustained load the progressive yielding is greater with an increased intensity of load. These characteristics occur at all temperatures but in general the higher the temperature, the greater the flow. This increase in yielding must be accompanied by a corresponding decrease in the modulus of elasticity. Another factor that has to be considered is that the coefficient of expansion of ice is variable and increases as the temperature rises. Thus the strain which the structure must resist depends not only on the time rate increase of the temperature of the ice, but also on the temperature at which the increase occurs, while the yielding of the ice governs the value of the modulus of elasticity to be used in translating strains into pressures.

Lofquist[35] obtained data of the same nature but used a considerably different experimental set-up in doing so. His apparatus consisted of a concrete vessel which was filled with water and placed in a freezing chamber such that the water was frozen in the same manner as would occur in nature. The heat transfer, when

cooled or heated, could take place only in the vertical direction. Gages to measure the pressure and temperature throughout the depth were placed in the ice. Also, thermocouples to measure the air and water temperature were used.

This experimental set-up permitted the obtaining of both a temperature and stress distribution throughout the depth of the ice sheet. From this data, it was then possible to plot a curve similar to the curve plotted by Brown and Clarke. This is shown on figures 1-2 and 1-3.

Although care was taken in an attempt to eliminate any effect of cracks resulting from the cooling of the ice, not all crack formations were eliminated and this was felt to have a reducing effect on the results. The concrete vessel in which the ice was confined would also expand or contract with changes in the temperature and consequently did not correspond to perfect restraint. The pressure in the case of perfect constraint was estimated to be up to 25% greater.

Comparing the curve of Lofquist with the curve of Brown and Clarke indicates a large deviation. Part of this deviation can be attributed to the lack of a uniform temperature gradient through the ice and uniaxial stress of the specimens from which the upper curve was obtained. An appreciable difference may also exist between different ice and, as Dorsey[14] indicated, even the direction of the orientation of the ice crystals can affect the rate of

yielding of the ice. It further points out some of the difficulties involved in trying to get a correct grasp of the ice pressure problem.

Lofquist also discovered that the ice pressure reaches its maximum in an ice sheet with a thickness around 20 inches. Thicknesses greater than this tend to have slightly lower maximum pressures developed or at best only slightly greater. This can be contributed to the decrease in rate of rise of temperature through thicker ice sheets. This effect compensates for the action of the increased thickness. The maximum ice pressure was also noted to occur when the mean ice temperature reached a value of 25°F. This might be a general situation or it might only be applicable to the particular nature of the tests performed. More work is necessary before it can be made a general statement.

A third investigation was made by Monfore[35] which was quite similar to the tests which Brown and Clarke performed. Cylinders 4 inches in diameter and 4 inches long were used. Thermocouples were placed in both the ice and air chamber so that both temperatures could be recorded.

One difference in running these tests was that, instead of a uniform air temperature increase, as the other investigators used, Monfore increased the ice temperature at a uniform rate. This required the raising of the air temperature at a greater rate for the first 15 minutes of the test. Depending on where the thermo-

couple for reading the ice temperature was placed in the test cylinder, the reading obtained could be high, average or low. This is due to the non-uniform temperature distribution which would be expected. Also, as with the work of Brown and Clarke, the cylinders had uniaxial restraint rather than biaxial as would be the case of ice in the natural state.

The curves which Monfore produced from his data were different than those which are plotted on figure 1-2. He plotted the maximum pressure as a function of the rate of temperature rise and also a plot of the time to reach the maximum pressure as a function of the rate of temperature rise. Assuming that the increase in pressure per hour is linear (Monfore found that it wasn't linear but that it didn't deviate much), it was possible to calculate a curve similar to the curves of figure 1-2. This was done by dividing the maximum pressure by the time required to obtain that pressure and changing the units from $\#/in.^2$ to k/ft^2. A plot is shown on figure 1-3.

The large deviation shown cannot be fully explained. One thing that must be noted is that for the upper curves the rate of temperature change is for the surrounding air while for Monfore's data it is the rate of temperature change in the ice. This would certainly be the reason for part of the deviation. Other causes can perhaps be contributed to the same reasons as were mentioned previously. At any rate such deviations only tend to

confuse the issues rather than help solve them. The interpretation of the data in calculating the values for the curve could also contribute to the deviation.

As can be seen, there is much work which is yet required before the relationship between the increase in pressure as a function of the rate of temperature increase is fully understood. However, the work done thus far has given a better understanding of the overall problem.

Once the relationship between temperature rise and increase in pressure is known, as well as the temperature gradients, it would be possible to predict for any ice sheet and any rate of temperature increase the maximum ice thrust that could be expected.

The temperature gradient can be assumed to be linear with the top surface at the temperature of the surrounding air and the bottom temperature at 32°F. as is shown in figure 1-1. Even if in actuality the temperature gradient was not linear, the error in assuming it so would not be expected to be very large. Further, assuming that the transfer of heat through the ice sheet satisfies the differential equation for heat flow, $\frac{\partial \theta}{\partial t} = h^2 \frac{\partial^2 \theta}{\partial x^2}$, the approximate method for calculating the heat flow as presented by Schmidt[37] may be used. This method was used by Rose[34] and has been shown experimentally by Lofquist[35] to be applicable.

This approximate method developed by Schmidt is essentially a finite difference solution of the differential equation for heat

flow. The thickness of the ice is broken into finite increments having equal thicknesses equal to ΔL. Then from equation 1-2, the value of ΔT can be calculated.

$$\frac{(\Delta L)^2}{2h^2 \, \Delta T} = 1 \qquad (1\text{-}2)$$

where:

ΔL is the increment of the slab thickness in ft.

h^2 is the diffusivity constant of ice in ft.^{2}/hr. (given as 0.0434 ft.^{2}/hr.)[14]

ΔT is the interval of time in hours.

The temperature gradients can then be calculated using the equation

$$\theta_2' = \tfrac{1}{2}\,(\theta_1 + \theta_2) \qquad (1\text{-}3)$$

where θ_2' is the temperature at the interval point 2 of the slab at the time interval ΔT and θ_1 and θ_2 are the temperatures at the interval points 1 and 3 of the slab at the start of the time interval ΔT. From the temperature gradient calculations, it is possible to calculate the average rate of temperature change for each interval. Then, knowing the relationship between the rate of temperature increase and rate of pressure increase from figures 1-2 or 1-3, it is possible to calculate the stress at these intervals for any increment of time. Summing up the stress over the entire ice thickness will give the total thrust for the particular increment of time.

Calculations of this nature were made by Rose[34] using the relation between temperature and pressure given by Brown and Clarke. Similar calculations are reported here using the relationships of both Brown and Clarke and Lofquist as are shown in figure 1-2. For these calculations the following assumptions were made:

1. The initial ice and air temperature is at $-22^{\circ}F$ and the ice temperature varies linearly through its thickness from $-22^{\circ}F$ to $32^{\circ}F$.
2. The temperature of the air and surface are the same at all times up to $32^{\circ}F$.
3. The ice remains a constant 18 in. thickness.
4. No absorption of solar energy.
5. Diffusivity constant of ice is 0.0434 $ft.^2/hr.$[14]
6. Uniform rate of temperature rise of $3^{\circ}/hr.$ at the top surface up to $32^{\circ}F.$ and then it is constant.
7. $\Delta L = 0.25$ ft.
8. $\Delta T = 0.72$ hr.

Forces after 5.04 hrs. and 10.08 hrs. were calculated. After 5.04 hrs. the force was 413 #/ft. using Brown and Clarke data and 2,822 #/ft. using the data of Lofquist, while after 10.08 hrs. the forces were 1,002 #/ft. and 7,262 #/ft. respectively. The large variation in the values as would be expected point out the need for more reliable data. However, once reliable data is available

it will be possible to predict the forces that could be expected using the theory presented. No attempt was made to compare similar calculations using the data of Monfore given in figure 1-3. This was partially due to the fact that it was impossible to calculate the relationship for the rate of pressure increase for values of temperature increase below 2°F/hr. using the data that was available. A much higher thrust than was calculated from the other two curves would have, however, been expected.

To be sure that the method of calculation used coincided with the calculation made by Rose, an independent calculation was made using his conditions. A comparison was then made from this calculation which verified the identicalness of the method.

Another variable which enters the problem that is not as significant in the expansion of ice against dams, is the elastic properties of the piers. A dam is generally rigid enough that the small amount of deformation that does take place is quite insignificant in relieving the stress. This is not necessarily the case of the elastic deformation of a bridge pier. To check whether or not the pier deformation could be important in relieving the stress, a hypothetical problem was set up.

A pier was assumed which is similar to the piers used for the bridge crossing the Yellowstone River at Glendive, Montana and is shown in figure 1-4.

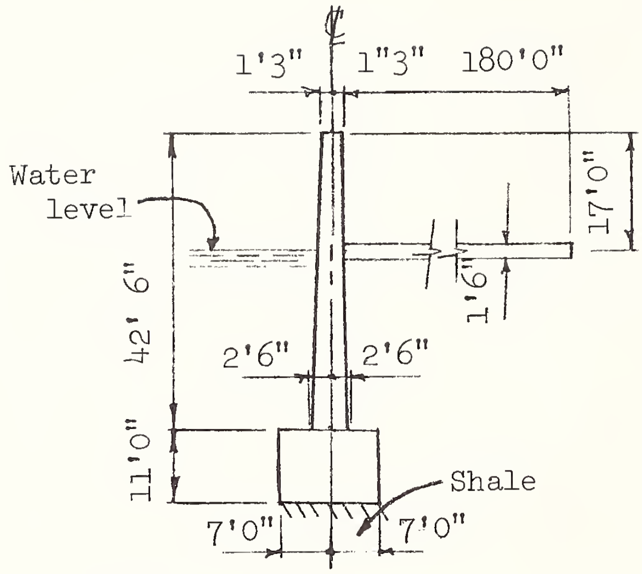


Figure 1-4

Typical Pier Section

Certain simplifying assumptions were necessary to make the calculations. First, a stiffness was assumed for the rotation of the foundation which was 1,650,000 $^{\text{k-ft.}}$/Rad.-ft. This was based on an elastic constant for the soil of 600 k/in./ft.[2] A comparison of this rotational stiffness of the foundation assumed to that which was given in a paper by Thomson et al,[38] although not fully applicable because of the difference in footing dimensions, indicated a close enough correlation to assume that the estimate is valid. Because of the uncertainty in real values to use for the resistance of the pier to bending and since the calculations were only to see if the deflection did affect the maximum pressure significantly, an average value for EI was assumed.

This value was 1.37×10^8 K-in.^{2}/ft. which was felt to be quite low and would represent an extreme condition.

Further, since no more reliable method is presently available, the expansion force of the ice was based on the elastic theory with minor modifications. The assumptions used in the calculations are as follows:

1. ΔT, the average change in ice temperature equal to 20°F temperature rise at mid surface.
2. α, the coefficient of expansion equal to 2.89×10^{-5} in./in./$^{\circ}$F.
3. Thickness of ice equal to 18 in. and remains constant.
4. L, the clear distance between piers equal to 180 ft.
5. Ice formed 17 ft. below the top of the pier.
6. Expansion of ice varies linearly from a maximum at the top to none at the bottom water surface.
7. The effect of the pier being pushed away from the bottom layers of ice due to expansion of the upper layers is neglected.

From the assumptions listed, it is possible to calculate the average expansion of the ice if it was unrestrained using equation 1-4.

$$\Delta_{ave.} = \alpha \Delta T L \qquad (1\text{-}4)$$

The average deflection, Δ ave., figured to be 1.25 in. Using

equation 1-5 and assuming that the maximum force that can be developed by ice expanding in the fully restrained case is 14 K/ft., it is possible to calculate an effective modulus of elasticity, E. The value of 14 K/ft. was arbitrarily chosen as an average value for the field tests done by Monfore.[35] The value for the effective E was calculated to be 112 K/in.^{2}

$$\Delta_{ave.} = \frac{PL}{AE} \qquad (1\text{-}5)$$

where:

P is the expansive force in K/ft.

L is the length between supports in inches.

A is the cross section area of one foot of width of the ice sheet in in.^{2}/ft.

E is the effective modulus of elasticity in k/in.^{2}

With this information, it is possible to determine the expansive force assuming elastic behavior of both the ice and bridge pier. An equation which equates the average deflection of the ice to the deflection of the pier at the point of loading both in terms of P was used. This is given as equation 1-6.

$$\Delta_{ave.} - \frac{PL}{AE} = \Delta \text{ pier} \qquad (1\text{-}6)$$

where:

$\Delta_{ave.}$ is the average unrestrained expansion of the ice in inches. (For this problem it was 1.25 in.)

$\frac{PL}{AE}$ is the deformation of the ice because of the restraint of the pier in inches.

Δ_{pier} is the elastic deflection of the pier caused by the expansion of the ice in inches.

Three conditions for the top of the pier were assumed which resulted in three values for the pier deflection. The first was the condition of no support. The second was assuming a pinned connection. And, the third was assuming a pinned support capable of carrying only one tenth of the expansive load.

The results of the calculation for the first condition indicated that a force equal to 35% of the completely restrained condition could be expected. For the second condition, the force was found to be equal to 90% of the completely restrained condition. The results for the third condition which is felt to be more typical for the actual bridge pier gave a value which was 80% of the fully restrained condition. Thus, under what is felt to be a normal condition for restraint the maximum decrease in pressure would not be expected to exceed 20%. However, if the top of the pier is constructed so that no lateral support can be expected, even greater reduction in the thermal force could result.

Actually, ice being a viscous solid, a much less reduction might be expected. The amount of flow of the ice is dependent upon the time as well as the magnitude of the load. A large load will cause a faster rate of creep than will a lower load. If the

deflection of the pier results in a lower initial load, the rate of further reduction in load because of creep will be slower. Thus, if this reduction in the rate of creep is sufficient to offset the decrease in force caused by the deflection of the pier, no reduction in the maximum force would be expected.

However, even if there was a considerable decrease in the expansive force of the ice caused by the movement of the pier such as was the case for the first condition, a force of 4.85 K/ft. could be expected if the maximum fully restrained force was 14 K/ft. The force would be 7 K/ft. if the maximum fully restrained force expected was 20 K/ft. Assuming a 35 ft. wide pier the total load would be 170^K and 245^K respectively which certainly would need to be considered in the design. Assuming the third condition of pier support and a fully restrained maximum pressure of 20 K/ft., the total load on the pier would be 560^K which would certainly cause severe cracking and possibly complete failure of the pier as it is presently designed. It should also be noted that the magnitude of the reduction in thrust caused by the deflection of the pier is also a function of the distance between piers. For the perfectly elastic case it would be directly proportional to the span distance. The shorter the distance between the piers the greater will be the reduction in the force due to pier deflection. Thus the percentages cited above are only applicable for the 180 ft. span assumed in the calculations.

Another problem that can accompany the formation of a sheet ice cover, is the pressure caused by fluctuations in the water level. An ice sheet freezing and adhering to the piers at one level will be broken by bending stress and crack when the water level drops. Freezing of the water between the cracks will result in a solid inverted arch or dome spanning between supports. A rise in water level can then cause considerable lateral thrust to be developed.

The magnitude of the ice pressure caused by variation in the water level depends on numerous factors. These include the curvature of the slab, length of the span, amount and rate of rise in water level, and physical properties of the ice. Because of the many possible variations in these factors, it is very difficult to obtain any general values or develop a general theory. For ice sheets of large span, the lateral thrust would not be expected to be very great but, with short spans, it is possible that the thrust could be as much as the thrust resulting from the thermal expansion. Lofquist[35] felt that in general the thrust caused by the change in water level would not be as great in many places as the maximum pressure resulting from temperature changes. However, he did feel that it could occur more frequently than the thrust resulting from thermal expansion. Certainly, in areas which were not subject to rapid changes in temperature, such forces could be quite important and should be investigated.

Before a general theory can be developed, a better knowledge of the plasticity property of the ice is necessary. Knowing this as well as the local conditions, it should not be too difficult to modify the present arch or shell theory sufficiently to be able to calculate the thrust resulting from water level variations.

Conclusions:

Whenever it is possible for a solid sheet of ice to form between piers, it is also probable that the piers will have to withstand some pressure resulting from thermal expansion of the ice. The magnitude of the force depends upon the initial ice temperature at the upper surface, rate of temperature increase, length of time for the temperature to increase, thickness of the ice sheet, any discontinuities such as open cracks, amount of insulation from snow, solar radiation, stiffness of the bridge pier and distance between piers.

It is not expected that the normal daily fluctuations in temperature will result in a critical thrust although some thrust can be expected. The maximum thrust is expected to result from rapid increases in temperature starting from a low initial temperature, and for a clear, snow free ice sheet approximately 20 inches thick with no discontinuities.

The effect of the deflection of the piers on the decrease in thrust would be greatest the closer the piers are together. Some

compensating for the deflection of the pier may occur because of a reduced rate of creep. Further investigations are needed to fully determine the effect of an elastic restraint on the maximum thermal force.

Before accurate calculations can be made which will predict the thrust from expanding ice, further research must also be undertaken in an effort to determine the relationship between the rate of temperature increase and ice pressure. Both laboratory and field investigations should be undertaken in this work. Further laboratory investigations of the nature performed by Lofquist[35] would be very useful. Also, field investigations which measure the pressure gradient, ice temperature gradient and air temperature all simultaneously and continuously should be made. With the more modern equipment available for the obtaining and recording of measurements, considerable improvement in the results should be possible.

Once the relationship between the rate of temperature increase and thermal ice pressure, the relationship between the maximum thrust and elastic supports, and the correlation between air-temperature and surface ice temperature are all ascertained, it will be possible to predict the maximum thrust that would be expected at any one site by knowing the maximum thickness of ice to expect, the maximum likely rate of air temperature rise and the extent of restraint. The maximum thickness of ice and rate of

air temperature rise can be determined, at least approximately, by a study of the meteorological records and site. The restraint can be determined from an anlysis of the piers and abutments. Sloping piers or abutments will give full restraint if the ice is frozen solidly to them.

Not all bridge sites are conducive to the formation of a smooth ice sheet nor are all piers at those sites which do offer conditions favorable for the ice sheet formation likely to be subjected to thermal expansion forces. Thus, only those piers which can be seen to be subject to thermal ice pressure need be designed for the anticipated load. This load can be expected to be as much as 20 K/ft. or greater depending on the local conditions. It might even be possible, at some sites where areas of the channel are favorable for the formation of an ice sheet, to change the channel sufficiently so that the velocity is more uniform and fast enough to eliminate the possibility of sheet ice forming. If so, no allowance for the expansive thrusts in the design would be necessary.

Thrust resulting from the variation in water level can also be expected in areas where an ice sheet can form between piers. A method for the prediction of the magnitude of the thrusts from this cause cannot be developed until the properties of ice under stress are better understood. Once the plasticity property of

ice is fully determinable, it will be possible to predict the thrust resulting from water level variations knowing the local conditions. Such thrusts are expected to be even more common than thrust from thermal expansion.

The buckling strength of ice is also dependent upon the plasticity property of the ice. However, if the buckling load could be accurately calculated, this load would be quite useful in establishing design criteria as it represents a maximum limiting value for the expansive force.

CHAPTER 2

FORCES ON BRIDGES RESULTING FROM MOVING ICE BLOCKS

The spring break-up of the winter ice cover on rivers lets loose large cakes of ice that may flow freely down the river at approximately the same velocity as the surface of the water. Such ice floes coming into contact with a bridge pier must undergo a change in velocity, direction, or both in either stopping or moving around the pier. This change in velocity or direction results in a force which must be resisted by the bridge pier.

A search through the literature turned up very little in the way of research on methods of predicting the magnitude of forces that could be expected. Only one paper[39] was found published in this country concerning the problem. The majority of work that is available appears to have been done in Russia. Two of these papers[40,41] were obtained and translated, and will be covered to some extent in the following discussion.

The type of pier that is most often used in regions where flowing ice is expected is of a rectangular shape running the full width of the bridge and extending either all the way up to the bridge deck or at least a height sufficiently above the high water line so that the columns framing into it are not subjected to moving ice loads. The upstream edge of the pier is designed

with an ice breaker pier nose which is formed at a 45° angle from the centerline with an 8 x 8" steel angle embedded in the concrete at the apex. The downstream edge is formed as a semi-circle. Either one or both edges may be vertical or sloped. A typical cross section is shown as figure 2-1.

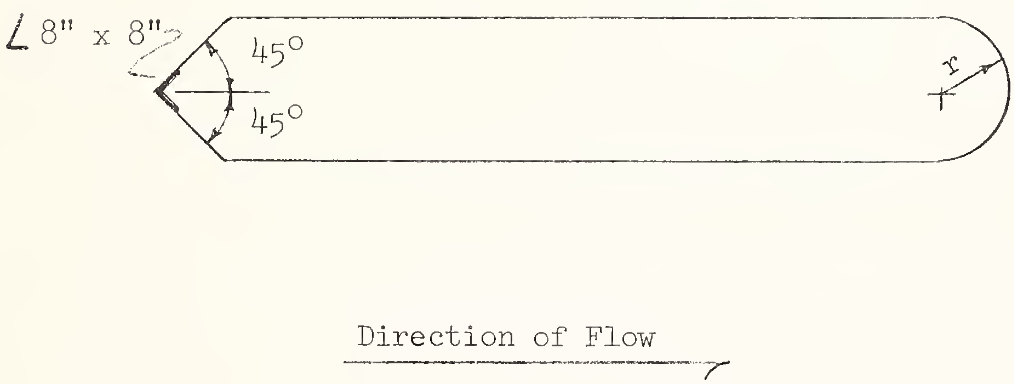


Figure 2-1

Typical Ice Breaking Bridge Pier Cross Section

The purpose of such a pier is to either split or turn the ice floes around it. A large floe striking the end of the pier will be crushed by the high pressure resulting from the sharp point of the upstream edge. Compressive stresses normal to the sloping side of the nose are developed as is shown on the freebody of a section of the ice sheet shown as figure 2-2.

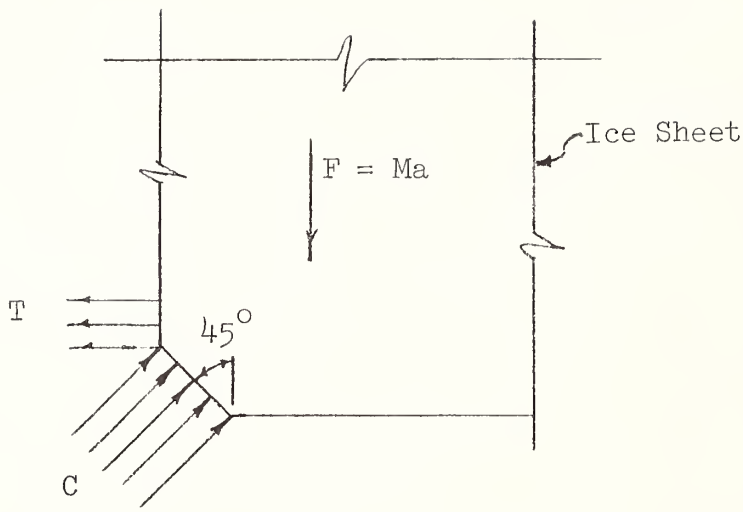


Figure 2-2

Freebody of Ice Floe Striking End of Pier

Resolving the compressive force, C, into two components, one parallel and one perpendicular to the long axis of the pier, it can be seen that a tensile force is developed at the apex which if equal to the tensile strength of the ice will cause it to split. The average and most widely accepted value for the ultimate strength of ice in tension is 100 $\#/in.^2$ [16] Using this value the theoretical splitting force per foot of ice depth would be 1200 #/ft. The forces perpendicular to the long axis of the pier would tend to cancel out and only a force parallel to the axis would be resisted by the pier. This force would be equal to twice the splitting force or 2400 #/ft. of ice thickness. The value given by Kirkham[39] was 2,200 #/ft. of ice thickness although the method by which he arrived at this value should have resulted in a value

identical to that which was developed above. The average thickness of such ice floes would not be expected to be greater than 3 ft. even under the most severe conditions. For such a three foot thick ice floe, the force exerted on the pier in splitting the floe would be only 7.2^K. A load of such a magnitude would have very little effect on the design of the pier.

A pier in which the front cutting edge is sloped from top to bottom splits the floe with a combined action of stress coming from two sources of loading on the ice floe. The first is the same as was described for the vertical pier face and the other occurs when the floe rides up on the edge of the pier causing bending stresses. The determination of the force exerted on the pier caused by this process is more complex although the magnitude would not be expected to be any greater than was previously developed.

After the ice floe is split, it is necessary for it to move around the pier. This results in an acceleration of each side and a resulting force against the pier. The average acceleration computed by Kirkham[39] was given as 2 ft/sec.^{2} Assuming a 100 ft. square ice floe 1 ft. thick split through the middle by the pier, he calculated the force required to move one half of the floe around the pier to be 18^K per foot of ice thickness normal to the side of the nose of the pier. If both sides move around the pier

simultaneously, the total force parallel to the long axis of the pier was given as approximately 26 K/ft. of ice thickness and there would be no resultant force acting perpendicular to the axis. If only one side moved around at a time the load would be equal to 13 K/ft. of ice thickness in both the perpendicular and parallel directions.

These values were obtained by using the equilibrium equation $F = ma$. For a 50 x 100 foot ice floe weighing approximately 290^k per ft. of ice thickness, and with an acceleration of 2 ft/sec^2,

$$F = \frac{290}{32.2} \times (2) = 18 \text{ K/ft. of ice thickness.}$$

Five theoretical methods for predicting the ice pressure on bridge piers with an ice-breaker nose, resulting from moving ice floes, are available for use in Russia. Two of these methods are described in the two papers[40,41] which were translated. Also in one paper[40] a report of the investigations made to measure the actual pressure on the bridge piers is given. The results of these seven investigations are compared to the results of the five theoretical methods for the same ice fields and are given in Table 2-I.[40]

Table 2-I gives the dimensions of the ice field as well as the observed and calculated values for the pressure. The first 5 ice fields come into contact with piers which had a sloping cutting edge while the last two piers were vertical. The first row of

	Ice Field	1	2	3	4	5	6	7
	Side of Pier	Sloping					Vertical	
	Area of Field (ft.^{2})	168,000	26,900	258,000	83,800	17,700	38,700	473,000
	Length of Field (ft.)	427	157	574	335	151	197	656
	Thickness of Field (ft.)	3.21	1.64	2.63	2.13	2.95	1.64	1.31
1	Pressure Observed (Kips)	101.0	38.6	102.2	99.9	47.0	138.0	121.2
2	T.U.P.M. - 57	335	172	276	227	309	408-611	393
3	G.O.S.T. 3440-46	670	344	552	454	618	393	377
4	T.U.P.M. - 56	33.8	14.8	29.1	19.4	34.9	370	355
5	Zeylor Method	448	121	282	201	364	252	243
6	Korzovin Method	160	56.2	120	123.5	146.6	203	195
7	Kirkham	1400	114.7	1762	464	135.8	165	1610

Table 2-I

pressures are the observed values followed by the five rows of the theoretically predicted values of the Russians. The last row is the predicted values as calculated using the equilibrium equation $F = ma$ and assuming the 2 ft./sec.^{2} acceleration given by Kirkham.

As can be seen, there is not much correlation between any of the methods. Actually, there is even some question as to the validity of the observed values since they were not obtained by direct measurement. The procedure used was to take motion pictures of the ice floe when it came into contact with the pier and moved by it. From these pictures, it was possible to determine the change in velocity as well as the time involved in splitting and moving the floe past the pier. By observation, the shape and size of the floe was estimated. The magnitude of load was then calculated knowing the maximum acceleration and size of floe using the principles of dynamics. Thus, the procedure seems to leave much latitude for there to be error present in the values given.

The method presented by Korzovin[40] appears to be empirical in nature although insufficient explanation and defining of symbols makes it impossible to get a thorough understanding of it.

The method presented by Gamaiunov[41] and given as method T.U.P.M.-56 assumed an ice field as a half infinite, elastic, isotropic, slab resting on an elastic foundation. Such a development ignores the fact that the ice is neither elastic or isotropic at

the time of ice cover break-up. Also, no consideration of the dynamic effect of the moving ice is taken into account. Both of these factors would contribute to the large discrepancies between the actual and theoretical calculations.

Other factors which could contribute to the large variations in the results are in the assumptions used for the strength of the ice at the time of motion, the manner in which the floes move around the pier, the velocity of the floes at the time of contact, and the acceleration of the floes after contact. These factors would certainly explain to some degree the large discrepancies noted when using the method presented by Kirkham[39] (the last row of values in Table 2-1). It cannot be expected that the acceleration for all cases would be the 2 ft./sec.^{2} as given. Other factors such as the time that it takes to split the floe, the orginal velocity of the floe, the velocity of the water around the pier, and the size and shape of the floe could all contribute to the rate of acceleration of the floe in moving around the pier after being split.

In general it can be said that the problem is a complicated one for which insufficient investigations have been made to define the many variables which need to be taken into account before a reliable theory can be developed. It is evident that much work is yet required in this area.

An attempt will now be made to more fully define the problem. Thus far, the only manner in which the ice floes have been assumed to strike the pier is directly in line with it. It has been further assumed that the floe will be split with each side moving around opposite sides of the pier. The load resisted by the pier is then parallel to the long axis and as such is quite easily carried.

Because of the great depth of the pier in the long direction, considerable error on the conservative side in determining the design load would contribute very little to the additional cost of construction. The resisting strength in the lateral direction is, on the other hand, much less and large lateral loads to be carried would tend to increase the cost of construction at a much faster rate. Thus, the determination of the actual lateral loads to be expected resulting from moving ice floes appears to be what is of major concern.

Three ways in which lateral loading on the pier could occur will be discussed. First, if the floe coming into contact with the end of the pier does not hit it squarely or toward the center of the floe, it is possible that, instead of splitting and moving around both sides of the pier, the floe could rotate and smash into the side of it as is shown in figure 2-3. Even if the floe did split and move out around each side, there would still be an unbalance in the lateral components of forces required to move the

ice around.

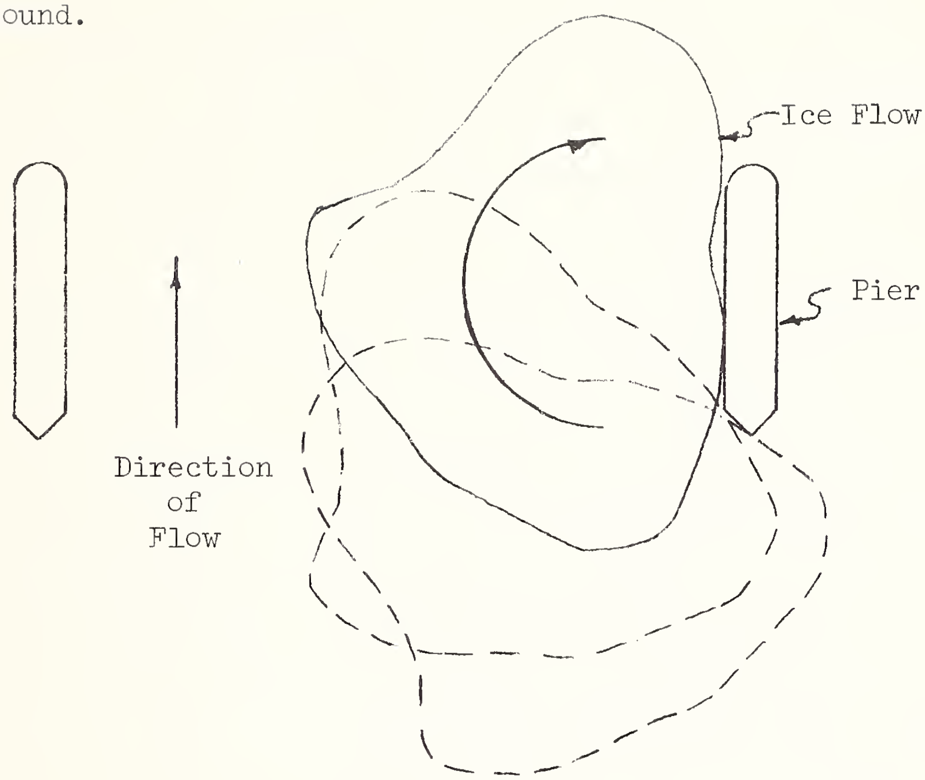


Figure 2-3

Floe Rotating Around Pier

The second way in which lateral loads may occur is for a very large floe to jam in between two adjacent piers as is shown in figure 2-4. Such an occurrence would result in a normal force on the sloping side of the pier sufficient to stop the floe or at least slow it down in which case the lateral component of the normal force could be critical.

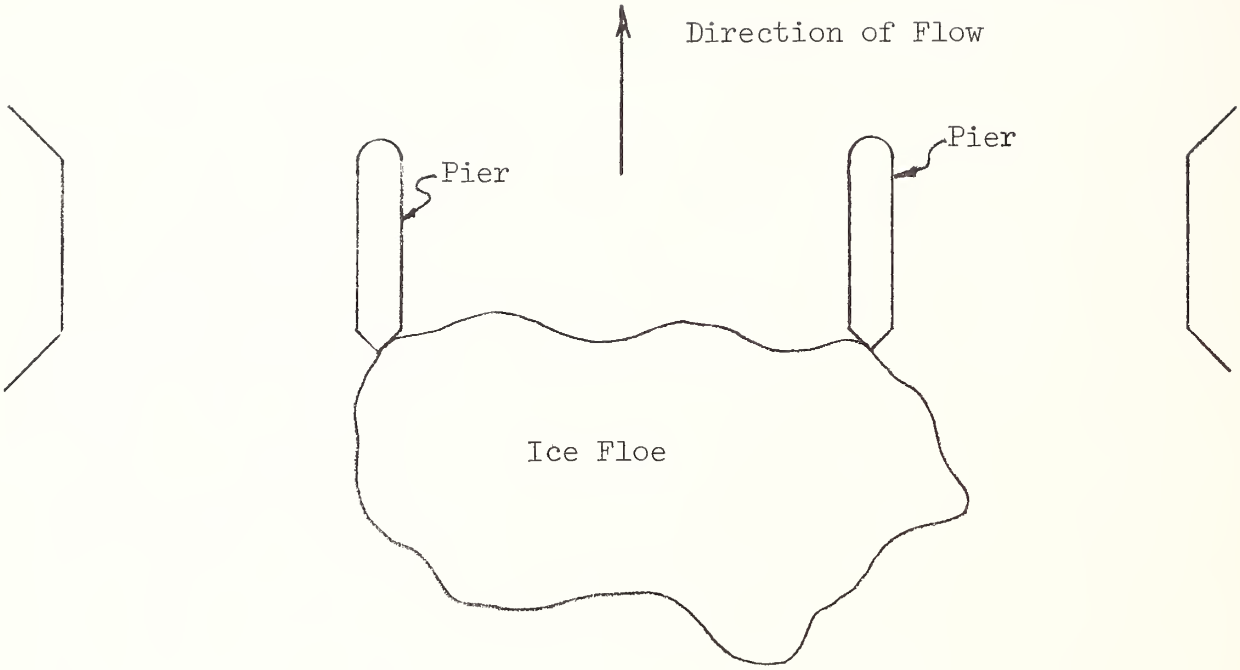


Figure 2-4

Ice Floe Jamming Between Piers

The third way of causing a lateral force, is for the floes to strike the pier at an angle. Such would be the case if the water flow is at an angle to the piers or if the direction of the floe was changed by its ricocheting off of an obstacle upstream from the bridge. Although the piers are generally placed parallel to the direction of flow when constructed, a change in the flow characteristics of the river could result in an angular flow of the water passing the piers. Also, a change in the direction of flow can be the result of ice blocking part of the channel. The latter situation would be expected to be more common.

The size of floes depends upon the type of ice cover on the river before the spring ice run. If the ice cover is a smooth sheet of ice, the thickness of the floes would not be expected to exceed three feet. The area for a floe as part of a smooth sheet of ice could be quite large and would depend upon the nature of the river and the way the cover breaks up. For large floes it would not be expected that they would double up one on top of the other so that the maximum thickness of ice coming into contact with the bridge would be the natural thickness.

An ice cover made up of moving ice jamming together could result in ice floes much thicker than the maximum three feet of smooth ice cover. The area of such floes would in general not be expected to be as large although an exception to this would be an ice gorge which could move out in one large floe. The tensile and compressive strength of such ice can generally be expected to be lower than that of smooth sheet ice.

Before one can predict the maximum force from moving ice floes it is necessary to be able to predict the types of ice cover which make up the floes as well as the maximum expected size. At the present time a field study would have to be made for each site as well as checking with local residents to determine past history of the nature of the flowing ice. With additional study on the formation of ice covers as well as a study on the correlation between

the type of ice cover and size of ice floes, it might be possible to predict the maximum size of floe to be expected. Until such time, however, only an educated guess as to the maximum size can be used.

Even if the maximum size and shape of an ice floe were known it would still be difficult, with the present knowledge, to predict the maximum force which could be expected for either the first or third sources of loading. More information is needed concerning the nature and causes of the velocity changes after the floes come into contact with the piers. The changes in velocity or direction of the flow would certainly be dependent upon the shape of the floe and the surface area which would come into contact with the pier. A sharp point on a floe contacting with the pier would tend to crush the ice and the rate of change in the velocity would be less than if a large surface of the flow came into contact where only elastic deformation takes place. With the deceleration being lower so also will the force be lower.

With impact, the pier can be required to absorb a large amount of energy or several times the load if applied statically. However, if the load can be assumed to be an impulse loading the maximum dynamic factor would be two, where the dynamic factor is defined as the ratio of the effective load to the load if applied statically. In both cases the effect on the pier depends upon the natural response of the pier to the loading.

Thus, before a method can be established to predict these forces, work must be undertaken to determine the nature of loading. Also, since the force appears to be a function of the velocity of the water, size of floe, direction of impact and strength properties of the ice, some correlation between these factors must be determined.

A maximum static load resulting from a large ice floe jamming between two piers can be calculated by assuming that the ice crushes against the sloping face of the bridge pier. This crushing of the ice against the side of the pier edge results in a lateral component of loading which can be shown to be quite large. Assuming a limiting value for the compression failure of the ice as 400 $\#/\text{in.}^2$, the average given in the literature,[16] and a width of the pier at the water level to be 3 ft. which gives a length of the sloping side equal to 21.2 in., the load normal to the sloping side is 122.0^K per foot of ice thickness. Resolving this load into the lateral direction gives a component of load equal to 86^K per foot of ice thickness. For a floe with a maximum thickness of 3 ft., the load would be $258,^K$ a ten foot thick ice gorge would cause a force of 860^K. However, it is not expected that the compressive stress of the ice in an ice gorge would be equal to the 400 $\#/\text{in.}^2$ assumed. This has also completely neglected the dynamic effect of ice when it becomes jammed which could considerably increase the load as given.

It was mentioned earlier in the paper that the velocity of the floes would be approximately the velocity of the water surface. The variation in this velocity would be the result of the effects of the various forces acting on the ice floe. These forces would be the gravity component of the weight of the floe acting on a sloping surface of the water, the wind force on the upper surface of the ice floe, and the hydrodynamic force of the water on the floe if the velocities are not the same. The gravity force would always act in the direction of water motion while the wind and hydrodynamic effects may act in either direction. These forces would also be acting on the floes when they came into contact with the bridge piers causing additional load. The effect is expected to be small and as such it might be possible to neglect them as individual quantities in a theoretical development. This would have to be checked in any investigation that is made into the overall problem.

Conclusions:

It can be expected that the load from moving ice floes striking a bridge pier is quite large. The resistance to this load by a typical ice breaking pier can be expected to be quite adequate in the long direction and even a very conservative value assumed for the ice load would not be expected to affect the present design or construction costs. The resistance to lateral

loads is much less and because of this a more accurate determination of the lateral load is necessary if an adequate design without costly overdesign is desired.

The magnitude of load depends to a greater extent on the size and velocity of the ice floes. Because of this, it can be expected that no two bridge sites would be identical nor would the anticipated maximum load be the same. However, knowing the maximum size of ice floes and the velocity of the river at the time of the ice movement, it should be possible to develop a theory which can predict the maximum force to expect from moving ice at any one bridge site.

To develop such a theory, it is necessary to make an investigation in an effort to obtain a better understanding of the nature of the loading and effect on the bridge pier. The determination of the maximum size of the ice floes should be possible with a better understanding of the mechanism of ice formation and a careful study of the site. Ice floes not to exceed 3 ft. in thickness could be expected in areas where a smooth ice cover can form. Much thicker floes could be possible in areas where ice covers form from jamming drift ice.

Although for the sample calculation the average value for the tension and compression of the ice was used, it is felt that these would be maximum values and that most of the time, if not all of the time much lower values might be expected. Certainly in the

case of floes made from jammed drift ice, the values would be found to be less. This is due to the softening effect of the warmer spring temperature. Investigations are necessary however if the actual values are desired.

Lateral loads may occur from three types of loading. These are the rotation of the flow around the end of the pier, an ice floe hitting the side of the pier from an angle, and jamming of the ice floe or floes between two adjacent piers. The jamming of the floes between the piers can only occur if the piers are fairly close together or if the floes are very large. It is, however, also possible for more than one floe to become jammed between the piers which would have the same effect as if they were one large one. In both cases the sloping sides of the front edge of the pier is the cause of the lateral load and as such would actually be detrimental to the resistance of the piers to ice loading.

Lateral loading resulting from an ice floe hitting the side of the pier or rotating around it can only occur if the river or at least a channel is carrying widely separated floes. If the river is packed solid with moving ice, the surrounding floes will keep the ice flowing in the direction of the river. Thus, only if the natural flow of the river is at an angle to the pier would large lateral loads be expected under this condition of flow.

In general it can be said that the problem is a very complex one for which a simple solution will not be available. Much work will be necessary before a reliable method for predicting the maximum forces can be developed and verified. For part of the work it is felt that some controlled laboratory investigation can be utilized but most of the work will require field investigations.

CHAPTER 3

FORCES ON BRIDGES RESULTING FROM ICE JAMS

Jamming of ice behind or under a bridge may occur either during the period of ice formation or spring break-up. In either case, the bridge or bridge piers may have to resist some or all of the force from such ice jams.

The formation of an ice cover during the winter ice season may originate at a bridge with the drift ice jamming between the piers to form the ice bridge necessary for the development of the ice cover. In such a case, it can be expected that almost the entire force of the ice cover must be supported by the bridge piers, at least during the time of early ice cover formation. If the conditions are such that a normal ice cover forms without any jamming as can be predicted by the theory on ice cover formation of Chapter 6, then knowing the channel cross-sectional properties, the velocity of water flow, and level of the water, it is possible to use the equations developed in Chapter 6 to calculate the thickness of the ice and resisting force of the ice at the bridge which must also be the resisting force of the bridge piers. The equation for calculating this resisting force is

$$R_i = \frac{\gamma_i t^2}{2} \left(1 - \frac{\rho'}{\rho}\right) \qquad (3\text{-}1)$$

where:

R_i is the internal resisting force of the ice in #/ft of width.

γ_i is the unit wt. of the ice in #/ft.^{3}

t is the thickness of the ice at the section in question in ft.

ρ' is the specific gravity of Ice.

ρ is the specific gravity of Water, equal to 1.

In order to see what magnitude of forces can be expected, a bridge will be assumed to be crossing the river which was assumed for the curves of figure 6-8 of Chapter 6. This river was 1200 ft. wide with an average depth of flow equal to 15 ft. The bridge will be assumed to consist of six spans at 200 ft. each from center to center of concrete piers. The top of the piers will be assumed to be 20 ft. above the 15 ft. water level. It is further assumed that the bank cohesion is 25 #/ft. and the water velocity is the maximum value that the ice cover can support with thickening by shove but with no jamming. From figure 6-8 the maximum thickness under these conditions is found to be approximately 5 ft. Substituting this value of t into equation 3-1 along with γ_i = 57.4 #/ft.^{3}, and ρ' = 0.92 the resisting force R_i is found to be 57.4 #/ft. and the maximum resisting load of each pier is found to be 200 times this value, or 11,480 #. Most bridge piers presently

being used would have little or no difficulty carrying a load of such magnitude.

If on the other hand the velocity increases so that jamming can occur, the depth to which the jam will go and the height of water level resulting from the jamming cannot be predicted by the present theory. However, if it is assumed that the water level raises so that the top of the ice is within 5 ft. of the top of the piers and jams down to within 5 ft. of the bottom, the total thickness of ice cover would be 25 ft. Assuming that the ice is still in a cohesionless state, the force per foot of width can still be calculated using equation 3-1. Substituting the value of 25 ft. into equation 3-1 for t, along with the same values for the unit weight and specific gravity of ice as before, results in a resisting force of the ice equal to 1,434 #/ft. If all of this force is assumed to be carried by the piers, each pier would have to carry 286.8^{K}. The significance of such a force in the design of bridge piers could be quite large depending upon the type of pier used, and as such, would need to be taken into account in the design.

In general, the force exerted on the bridge piers resulting from the normal thickening of the ice as predicted by the theory of Chapter 6 would not be expected to be too significant, although it should be taken into account in the design. The force exerted

on a bridge by an ice cover which has resulted in a hanging dam or ice gorge would be expected to be much larger. No method is yet available to fully predict the extent of thickening for such a case and until a method is available any estimate must be made by assuming a thickness for the cover as was done in this paper.

The validity of using equation 3-1 should also be questioned until it can be proven to be correct. This would require further investigation into the phenomenon of the ice formation as is mentioned in Chapter 6.

Bridges which are located across stretches of river which may develop hanging dams or ice gorging are further endangered by the build-up of the cover extending up to the underside of the bridge deck. Since most bridge decks are not generally designed to resist either horizontal or upward forces of much magnitude, their resistance to this type of loading is generally quite low. Thus, an ice cover built up to the bridge deck could cause considerable damage from any upward or lateral force which the ice might exert against it.

The magnitude of such forces, even if there was a method available to predict the extent of ice build-up, would be very difficult to determine. Actually, it would not be at all desirable to have the ice cover forcing against the bridge deck since even if the main members and connections were adequately designed to carry the expected load, considerable damage could and

would occur to small secondary members such as bracing. Therefore, the modification and extension of the theory presented in Chapter 6 to include a method for predicting the height of build-up of hanging dams and ice gorging would be very useful for the determination of the height of the bridge necessary to clear the ice under the most severe conditions of ice build-up.

When the warm days of spring come, the large quantity of ice which has formed over the river becomes weakened and, with the rise in water level due to the increased flow from melting snow and ice, will break up and begin to move. This is the start of the spring ice run. Such moving out of the ice can go quite smoothly and within a day or two the river will be completely open with only the sheared edges of ice on the banks of the river as a reminder of the time when the entire river was closed. Generally, however, the ice does not go out smoothly but instead it becomes jammed and causes ice shoves and flooding.

As in the case of an ice cover formation from drift ice, any constriction in the river can cause jamming of the moving ice. Some rivers have particular stretches which will cause jamming of the ice each spring. The two most common sources of jamming during the spring ice run are the narrowing of the river and ice covers down stream which have not gone out prior to the ice upstream. The latter cause is generally the harder to predict

since it is related closely to the meterological conditions along the stream which can vary considerably from year to year. Thus, if the warming of the river begins at the lower end and proceeds upstream, little trouble would be expected since the cover below would generally go out first. On the other hand when the upper reaches of the river are warmed prior to the lower parts, considerable jamming can occur when the ice from above comes into contact with the ice cover still remaining below.

Often times it is possible, from past observations, to be able to determine where jamming of the moving ice can be regularly expected. Many other areas that don't generally cause jamming of the ice could do so under more adverse conditions and such locations would be quite difficult to determine. Even a bridge itself could be the cause of the ice jamming in certain instances to be discussed. Therefore, some provision should be made in the design of a bridge to account for the possibility of an ice jam occuring in the vicinity of the bridge resulting in a force acting on the bridge piers.

Not only a knowledge of the location of the places which regularly cause jamming but also a knowledge of the height of damming of the water is necessary if safe guards are to be made in the location and design of a bridge. For some rivers, Barnes[16] notes that the highest water levels and flooding occur in the winter during the ice season or in the spring during the ice run

rather than during the time of peak spring runoffs or flash flooding. A case of high water during the spring ice run of 1963 is shown on figure 3-1. If the water level had gone slightly higher, the ice would have come into contact with the bridge girders and possibly resulted in a jam and damage to the bridge. This increased stage and flooding may, however, be of a local nature where only a short stretch of the river raises in stage and floods. In such a case, unless a stream stage recorder was located in the area of flooding, no official record of it would be available. Generally, however, residents of the area would be available with sufficient knowledge of the jamming and flooding to enable the designer to adequately account for it in his design.

When a bridge is located in an area where jamming of ice can be expected, three conditions of loading are possible. The first condition is when the piers of the bridge restrict the flow of ice and cause jamming. The second is the catching and holding of the ice by the bridge piers and deck system. And, the third is the lateral force on the bridge pier resulting from an ice jam extending through at least one span while the adjacent span is clear. This is shown in figure 3-2.

Figure 3-1. Ice Run on the Missouri River. Townsend, Montana, 1963

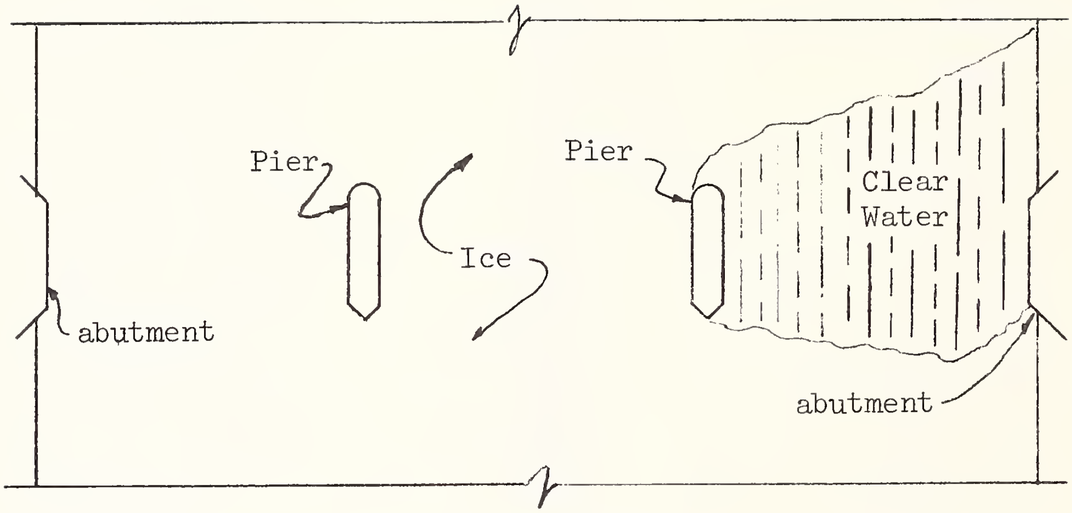


Figure 3-2

Condition for Lateral Pier Loading

At the present time there is no available theory for the predicting of the formation of an ice jam, the extent of jam build-up, or the forces which a jam could exert against a bridge or other obstacle during the period of the spring ice cover destruction. It is expected, however, that the same principles that are applicable for the prediction of ice cover formations from drift ice, Chapter 6, may also, with some modification, be applicable here. The assumption of a cohesionless material would still be applicable even though the size of pieces, making up the jam, are in general much larger.

There are two places where there appears to be a need for a possible modification in the theory presented in Chapter 6. One is the assumption that the angle of internal friction is equal to zero. For the size and shape of the block expected at this

time, it is felt that there would be some internal friction to help resist the forces acting on the jam. Since the internal resisting force of the ice cover is the same as must be resisted by the bridge piers, the load on the bridge piers will also be increased because of the angle of internal friction.

The other modification is in the values of the unit weight and specific gravity that should be used. Considerable water filled voids are expected to occur in any jam or cover made up from the broken pieces of old cover, therefore higher values for the unit weight and specific gravity are expected. Thus before any theory so developed can be used with confidence, the values for the angle of internal friction, and specific gravity would have to be checked along with the check on the general theory.

Actually, the only equation that would have any significant change would be the equation for the internal resisting force given as equation 3-1. This equation must be modified to take into account an angle of internal friction which has been assumed in the following development.

By taking a small prism in equilibrium out of the ice jam (figure 3-3) and applying Rankine's theory regarding the behavior of a granular material in the active state, the equation for the internal resisting force can be calculated.

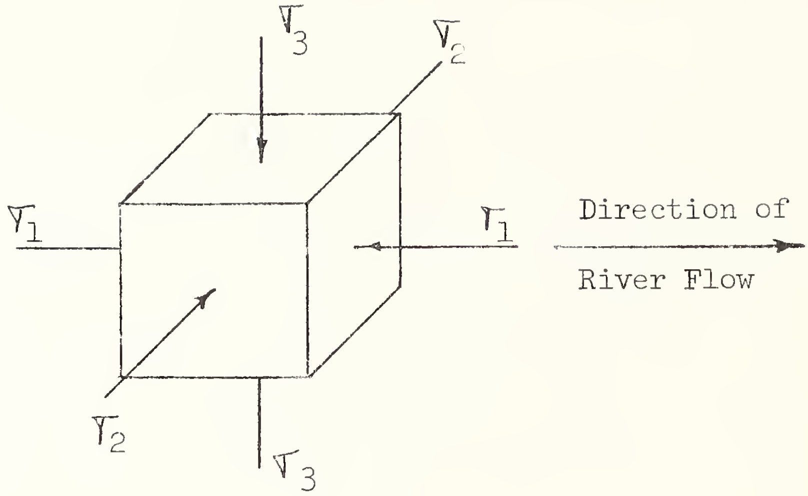


Figure 3-3

Differential Stress Block

The relationship between the stresses is given by equation 3-2.

$$\sigma_2 = \sigma_3 = \sigma_1 \tan^2 (45 - \frac{\phi}{2}) \qquad (3\text{-}2)$$

where:

σ_1 is the stress in the direction of the current in $\#/ft^2$.

σ_2 is the stress perpendicular to the direction of the current in $\#/ft^2$.

σ_3 is the stress perpendicular to the water surface in $\#/ft^2$.

ϕ is the angle of internal friction in degrees.

Since the ice would be expected to have a random orientation in the jam, σ_2 would equal σ_3. This was verified for the case of pulpwood jams[30] and, because of the similarity in the nature of the jams, it is expected to be true for ice also.

The stress σ_3 can be calculated if it is assumed to vary with the depth and then the pressure at the water line resulting from the weight of the ice above must be equal to the buoyant upward force of the displaced water below. (See figure 3-4) For such a case, $\sigma_{3\ max.} = \gamma_i a$

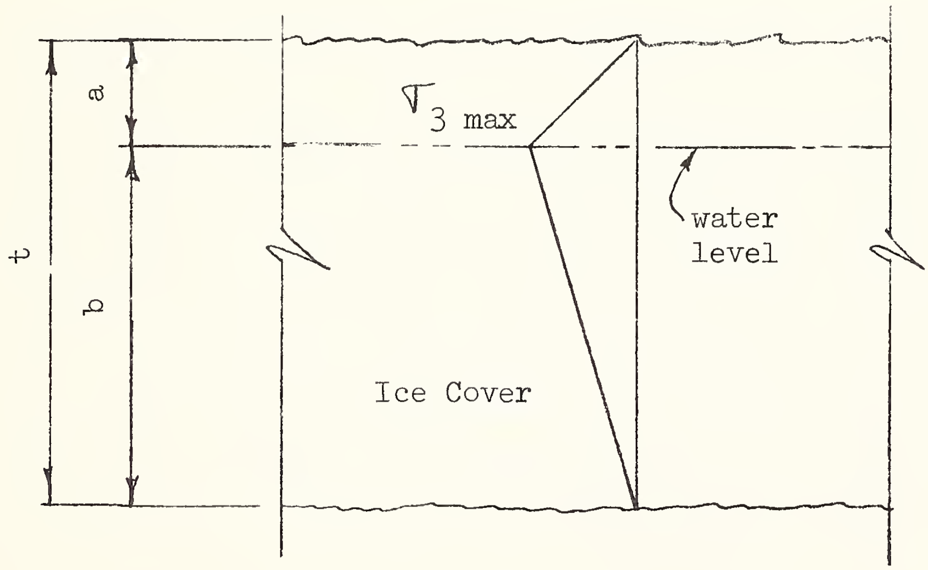


Figure 3-4

Stress Distribution in the Ice Cover

where γ_i is the unit weight of ice in #/ft.^{3} and a is the distance from the top of the jam to the water level in ft. The total force, F_3, can then be calculated by summing the stress over the full thickness, whereby $F_3 = \sigma_3 \frac{t}{2} = \gamma_i\ a\ \frac{t}{2}$. Eliminating a as is done in Chapter 6, results in the equation, $F_3 = \gamma_i \left(\frac{\rho - \rho'}{\rho}\right) \frac{t^2}{2}$.

However, since the force is directly proportional to the stress,

$F_3 = R_i \tan^2 (45 - \frac{\phi}{2}) = \gamma_i \frac{(\rho - \rho')}{\rho} \frac{t^2}{2}$ where R_i is the internal resisting force of the ice jam in #/ft. of width. A rearrangement of the above equations results in equation 3-3.

$$R_i = \gamma_i \frac{(\rho - \rho')}{\rho} \frac{t^2}{2} \tan^2 (45 + \frac{\phi}{2}) \qquad (3\text{-}3)$$

Using the relationship of equation 3-3 and the theory of Chapter 6, it should be possible, knowing the values to use in the equations, to predict whether a normal cohesionless cover would be expected to form behind the original ice bridge or if jamming from shove would take place. If the velocity of flow is such that jamming does not occur, assuming no bank cohesion, then the present modified theory can be used to predict the thickness of cover and the force which must be holding it in equilibrium. On the other hand if the velocity of flow is such that jamming is expected to occur, there is no way as yet to predict the depth of jamming and rise of water level.

Because of the uncertainty in the validity of the theory and the values of the constants to use in the equations, no further work was undertaken to complete the modification of the equations of Chapter 6 into a general form to use in these cases. However, calculations have been made by assuming values for t and ϕ for equation 3-3 to cover the first condition of loading where the bridge piers or abutments restrict the flow of ice and cause it to jam. For

these calculations the same channel cross sections and bridge pier spacing will be used as before. That is, a 1200 ft. wide channel and a 200 ft. spacing of piers. Assuming that t = 25 ft, $\phi = 20^\circ$, $\rho' = 0.92$, and $\gamma_i = 57.4$ #/ft.^{3}, from equation 3-3, R_i is equal to 2,930 #/ft. If it is assumed that the piers carry all of the load, this load would be 586^K per pier. Such a force would certainly be significant in the design.

If the coefficient for internal friction had been assumed to be zero the force would be 286.8^K the same as was calculated for the case of jamming during the ice cover formation. The effect of the angle of internal friction on the increase of the internal resisting force can be seen in figure 3-5. This points out the need for an investigation to determine the correct value if reliable calculations are to be made using equation 3-3.

An example calculation for the second condition of loading, which is the jamming of the ice behind the bridge deck and piers was not made. This was justified in that the load distribution between the piers and deck would be dependent on the angle of internal friction, which at this time can only be a guess. The smaller this angle of internal friction, the larger is the percentage of load that must be resisted by the bridge deck, while at the same time, the piers will be required to carry less. Also, since it is felt that the bridge deck should clear any jam that might

ANGLE OF INTERNAL FRICTION VS. INCREASE IN INTERNAL RESISTANCE

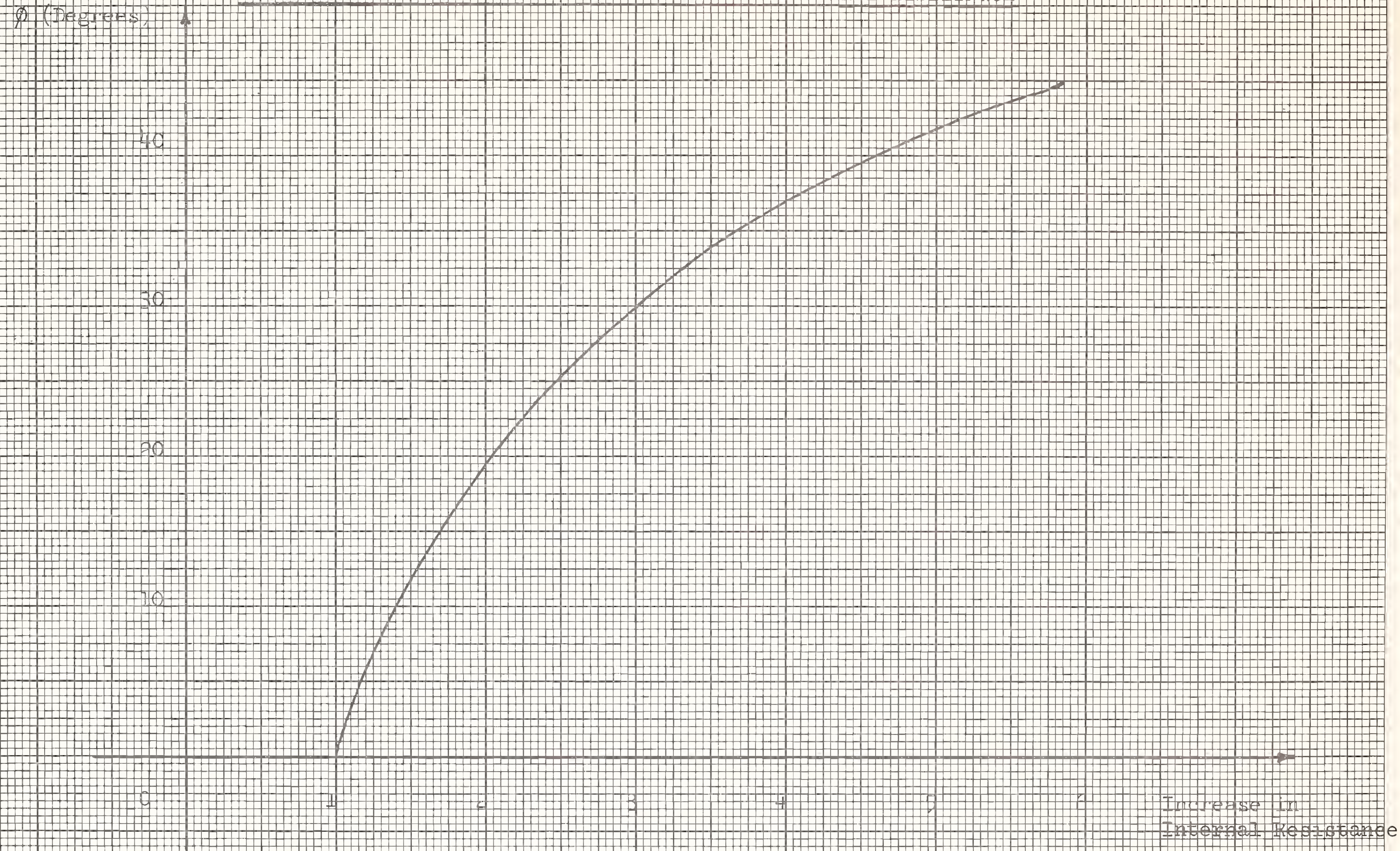


FIGURE 3-5

occur and should not be required in the design to carry any load resulting from an ice jam, there seems to be no need to pursue this course any further.

An example is presented for the third condition of loading (figure 3-2) which is a lateral force against the pier resulting from the ice jam extending through at least one span under the bridge while the adjacent span is clear. For this case, it is necessary to refer back to the relationship $\sigma_2 = \sigma_3$ from which the equation $F_2 = F_3 = \gamma_i \frac{t^2}{2}\frac{(\rho - \rho')}{\rho}$, which is the same as for the case of no internal friction, can be developed. Substituting into the above equation for F_2 the values of γ_i equal to 57.4 #/ft.^{3}, t equal to 25 ft. and ρ' equal to 0.92 gives a value of F_2 = 1.434 #/ft. of pier length. Estimating the pier to be 35 ft. long the total load would be 50.2^K which is sufficiently large that it should not be overlooked in design if it is possible that such a load could exist.

A constant value of 25 ft. for the thickness of the ice jam was used for the examples of calculations under the different conditions of the ice forcing against a bridge pier. In order to see the difference in the resisting force necessary to keep the ice jam in equilibrium as a function of the thickness, a plot is given, figure 3-6.

The curve of figure 3-6 is calculated assuming that ϕ is zero. The value for the resisting force when ϕ is not zero can

THICKNESS OF ICE VS. INTERNAL RESISTANCE OF ICE

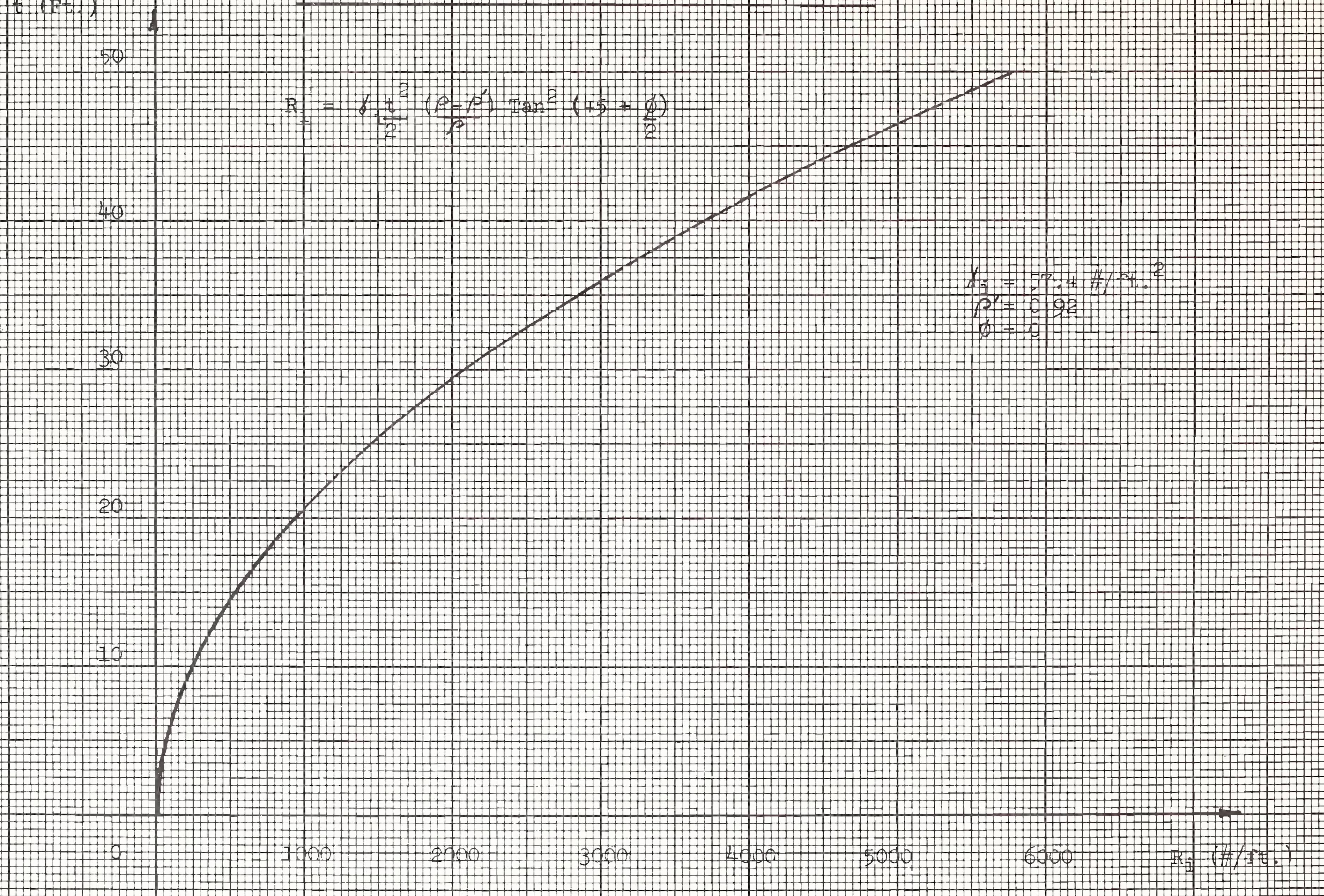


FIGURE 3-6

be obtained by determining the factor for the increase in resistance from figure 3-5 and multiply it by the value of the resistance of ice when ϕ is zero obtained from figure 3-6 for any values of t and ϕ desired. This will give the force per unit width of river that must be resisted by any obstruction that is holding the ice in equilibrium. Part of this resistance would be supplied by the banks. However, it is not expected that more than one-half of the force from the span adjacent to the banks would be carried by them. Therefore, the forces calculated for the piers should not need to be adjusted in any way to account for the bank friction. This is of course only a supposition and further investigation could find it in error.

As can be seen from figure 3-6, R_i increases as the square of t. Therefore, for every doubling of the thickness the force required to hold the jam in equilibrium goes up 4 times. This is also assuming that, if there is an angle of internal friction, it is a constant. Actually, it is quite possible that the angle of internal friction increases with the thickening of the jam in which case the doubling of the thickness would result in a multiplying factor greater than 4.

Because of the quadrupling factor, it is important that the maximum predicted or effective jam thickness be used in the calculations. Since at the present time there is no theoretical

means of predicting the thickness of a jam with certainty, the effective thickness should probably be used if the bridge site has a history of ice jamming. This effective thickness can be determined by the two boundaries which are the bottom of the river and top of the normal river channel. If the ice is jammed to the bottom, it will be in part resisted by the friction between the ice and bottom of the river and as such would tend to relieve some of the force carried by the structure. And, if the ice jams to a height exceeding the height of channel, flooding will occur and the river will be unable to rise sufficiently to build a deeper dam. Therefore, it appears as if the maximum effective jam would occur when it had extended down to within a short distance of the river bottom and up to a height slightly above the level necessary to cause flooding.

There are, however, reports[16,25] of ice shoves forcing ice into large piles along the banks, some as high as 30 ft. Such occurrences at the present time would be hard to predict and guard against. If an area has a past history of ice shoves of this nature, it might be wise to choose another site for the bridge where this is not a problem.

Conclusions

Before it is possible to predict with any accuracy the forces resulting from ice jams that can be expected to be resisted by a

bridge, it is necessary to establish criteria for the prediction of the extent of ice cover build-up or jamming which can occur either during the winter ice cover formation or spring ice run. The relationships presented in this discussion as well as in chapter 6 require verification before they can be put to general use. As future investigations are made, it might even be found that the theory so presented is not fully applicable and new relationships will have to be developed. Even so, what has been presented is a start upon which future studies may be developed.

The need for future study, in addition to that which is mentioned in the conclusion of Chapter 6, includes an investigation to determine the angle of internal friction and specific gravity of an ice jam forming at the time of the spring ice run. For such studies it might be possible to get some data from a laboratory study which of course would be much simpler than the field studies.

An important factor to take into account when designing a bridge is the height. Care should be exercised to see that the bridge is situated well above the level of the ice jam or cover. At the present time, the best way to determine the necessary height of a bridge in places where ice covers or ice jams form is to interview the local residents and physically check the site. In the future with additional work on the problem, theoretical relationships should further aid the designer in locating and establishing a height for such bridges.

Only the first and third conditions of loading mentioned previously would be expected as long as the bridge is located sufficiently high so that the ice does not come into contact with the deck. Thus, it is not necessary to make an investigation to ascertain the proportion of loading that could be expected to be carried by the bridge deck.

In general, it would not be expected that a bridge, located in a stretch of river that had no previous history of jamming, would cause jams to form except in unusual circumstances or if the bridge was not situated high enough to permit the free flow of the ice. It is not always possible to locate a bridge in areas where jamming of the ice is not a regular occurrence, in which case the possibility of jamming should be taken into account in the design. Even in areas which do not have a history of jams, if the end abutments are placed far enough into the river channel, a condition sufficient to cause jamming could be developed.

The calculations made using the present unverified theory can at best be considered quite crude. They do, however, give a rough estimate as to the possible range of loading that could be expected. Until something better is developed and the theory is verified, the use of the equations should only be made with the idea of arriving at some estimate of the possible loading which is at least more than has been available in the past.

PART II

CHAPTER 4

PHYSICAL PROPERTIES OF ICE

Ice is a crystalline substance that is generally accepted as being elasto-plastic where the elastic limit is very low. However, Butkovich and Landauer,[9] having measured creep rates for ice in uniaxial compression at very low pressures, indicate that for all practical purposes ice can be assumed to be plastic. Under sustained load, Jellinek and Brill[10] found that the Newtonian flow of ice only occurred in stress regions below 30 $\#/\text{in.}^2$ and is given as equation 4-1.

$$\tau = \eta \frac{d\gamma}{dt} \qquad (4\text{-}1)$$

where τ is the shearing stress, η is the coefficient of viscosity, and $d\gamma/dt$ is the rate of shear strain. At higher stresses, Butkovich and Landauer[11] found that the minimum creep rate was not a linear function of stress. They found that, although there was a considerable scattering of experimental data, a power law was the best equation for satisfying the data as a relationship for the minimum creep rate. Such an equation is given as equation 4-2.

$$\frac{d\gamma}{dt} = k\tau^n \qquad (4\text{-}2)$$

where constants k and n were given as 0.863×10^{-8} and 2.96 respectively as an average of all tests, excluding single crystals oriented for easy glide. There was sufficient deviation between

the data for the different sources of ice to question the validity of one set of coefficients to satisfy all ice. Also, it might be found that the rate of creep is a function of temperature which did not appear to be taken into account in this work.

Although ice is primarily a plastic material, Butkovich[12] indicated that for the cases of very rapid loading, or low loads for short duration, it can be treated as an elastic material.

Another problem in obtaining physical properties for ice is that it is rarely found in the pure state in nature. Impurities such as dirt, air pockets, and soluble materials may be found in ice; all of which tend to vary the physical properties.

Not all ice is formed the same way and this also can effect a variation in physical properties. For example, an ice cover made from moving ice is a conglomerate of all forms of ice as well as snow which has frozen together and tends to be granular in texture. Smooth ice, such as that which forms on lakes, has grains which are usually elongated in the direction of freezing.

The properties of ice also tend to be somewhat dependent upon the temperature of the ice. Although, at the present, the actual relationship for all properties has not been established.

Further, standard testing procedures have only recently been established[13] which, for most of the data available, means that arbitrary techniques have been chosen. The size, type, and shape of specimens, rate of loading, and techniques of the various

investigators have varied considerably, contributing to the wide range of values given for the physical properties in the literature.

All of the important work that has been done in the field of obtaining the physical properties of ice up until 1950 is compiled in two publications. The first is a book by Dorsey,[14] and the other is a publication edited by Mantis.[15] Since that time much work has been done by the Snow, Ice, and Permafrost Research Establishment, a division of U. S. Army Corps of Engineers, in an attempt to establish reliable physical constants. Therefore, only what are considered the more reliable average values for some of properties of ice will be given here.

Both static and dynamic methods have been used to determine the Modulus of Elasticity, but only the dynamic methods are held to be reliable. Both Dorsey[14] and Mantis[15] consider the values they list from the work of Boyle and Sproule to be the most reliable. The average value for E of this work is $1.396 \times 10^6 \#/\text{in.}^2$

The source of the ice seems to have a considerable effect on the ultimate tensile, compressive, and shear strengths. The mean value assumed for the compressive strength of smooth river ice has been $400 \#/\text{in.}^2$ with test values averaging $417 \#/\text{in.}^2$ for Barnes[16] and $440 \#/\text{in.}^2$ for Weinberg.[15] For lake ice, Butkovich[12] gives a range of compressive strengths varying from 500 to $800 \#/\text{in.}^2$

The tensile and shear strengths of ice are found to be approximately the same and, for river ice, they have been assumed to be

100 $\#/in.^2$ with average test values given as 103 $\#/in.^2$[16] and 90 to 95 $\#/in.^2$[15]

The most reliable value for the coefficient of expansion of ice is given by Jacob and Erk,[14,15] and is

$$\alpha \times 10^6 = 52.52 - 0.1852t + 0.00885t^2 - 0.000237t^3 \text{ in./in.}^\circ C$$

where t is the temperature in centigrade.

Other physical properties of ice have been determined but will not be listed here. For additional information, the reader is referred to the references mentioned earlier.

Because of the conglomerate nature of much of the ice which is formed on the rivers and streams in this region, additional information is going to be needed on the physical properties of river ice before reliable theories for the prediction of forces of ice against bridges can be developed. The physical properties of ice which are needed include values for the compressive, tensile and shear strengths, bank cohesion, the coefficient of friction, modulus of elasticity, and creep rate. These values must be obtained at all stages of the ice formation and destruction. Also needed is reliable data on the rate of pressure increase as a function of the rate of temperature increase for the various types of ice formed. The use of the standard methods of testing to determine the ultimate strength values presented by Butkovich[13]

should aid in obtaining consistent values and will permit comparison of results without fear of having discrepancies caused by variations in testing procedures.

CHAPTER 5

MECHANISMS OF ICE AND ICE COVER FORMATIONS

The cooling of the water and bed of the rivers by radiation, conduction, and convection in the fall of the year eventually brings the temperature of the water to the freezing point. At this time, depending upon the velocity and turbulence of flow, one or all three of the forms of ice may develop. These three forms of ice are referred to in most literature on ice as sheet, frazil, and anchor ice.

Sheet ice forms on the surface of lakes, pools, and very slowly moving rivers. The manner in which it forms has been described by Stevens,[17] Parsons,[18] and Barnes.[19] Ice crystals begin to form and spread out over the surface toward the center from the water's edge. In order for these crystals to form, there must be available some nucleus. These nuclei may be tiny globules of dissolved air or any other foreign matter which might be in the water at the time of freezing. Once the surface of the water is frozen over the continued thickening of the cover is at a much slower rate which decreases as it thickens since the water under it must be cooled by conduction of heat through the cover. Any snow on the ice cover will also decrease the thermal conductivity and thus reduce its continued thickening. The amount of heat transfer from a body of water and the thickening of the ice sheet can be

predicted by present theories of heat transfer. The most recent work on this topic was presented by Baines.[20]

However, Parsons[18] notes an abnormal phenomenon of the growth of sheet ice in that the rate of thickening was noted to decrease at an abnormally rapid rate when a prolonged cold spell with temperatures remaining continuously below $0^{\circ}F$ persisted. Whenever the temperature fluctuates enough to approach the freezing point a normal rate of thickening occurs. The explanation given for this is that liquid water is supposedly a mixture of several molecular forms of H_2O with only one which can change into ice. At normal temperatures, the water maintains normal proportions of the molecular forms by a continuous interchange of form but, as the freezing point is reached, the rate of interchange decreases. During a prolonged cold spell, the ice forming molecules are rapidly used up, thus reducing the formation of new ice. The fluctuating temperature, on the other hand, permits the maintaining of a normal proportion of the ice forming molecules.

Once an ice sheet has formed over a body of water the temperature of the water will remain slightly above freezing except at the interface between the water and ice. This prevents the further formation of underwater frazil and anchor ice. By utilizing the fact that underwater ice which causes trouble with hydroelectric facilities cannot form under an ice cover, engineers have been able to design the upstream areas of a plant so that an

ice cover will form early in the winter and thus eliminate much of the ice problems throughout the rest of the winter.

Supercooled water which is vigorously agitated, such as a turbulent stream, will tend to freeze into crystals known as frazil ice. The degree of supercooling required for frazil ice to form is given as not generally exceeding $-0.01^{o}C$[21], although it is stated that the air temperature must be much cooler.

There appears to be some question as to whether or not frazil ice forms throughout the depth of the water or only on the surface. Parsons,[18] Schaffer,[21] Stevens[17] and Barnes[19] all say that frazil ice is surface-formed ice which is carried down into the water because of the turbulence. Altberg[22] and Barnes,[16] changing his opinion in his later book, state that the frazil ice may form throughout the water providing that it is sufficiently supercooled. Part of this difference in opinion might be contributed to the fact that it was earlier held that the temperature of a river was uniform throughout its depth. Bydin,[23] however, measured the temperature gradient in larger rivers and found it to vary considerably. Thus, one observing the formation of frazil ice on a large river in which only the surface is supercooled would come to the conclusion that it only formed on the surface. Others observing frazil ice formation on shallower and more turbulent rivers where the water temperature is supercooled throughout its depth would conclude that frazil ice forms throughout the river depth. Therefore

it appears logical to conclude that frazil ice forms whenever the water becomes supercooled regardless of how deep this may occur in the river.

Schaffer[21] describes frazil ice as a thin, free-floating, round disk. These particles are 1000 to 5000 microns in diameter and are 25 to 150 microns thick.

The frazil ice particles, although less dense than the water, can be carried down by the turbulence and have been known to be found as deep as 80 to 90 feet.[16, 22] The buoyancy of frazil ice in water is dependent upon the size of the particles and the viscous resistance of the water. Barnes[16] gives the viscous drag on a small round body as $f_v = 6\pi\mu rV$ where f_v is the viscous drag, μ is the coefficient of viscosity, r is the radius of the particle, and V is the terminal velocity. He also gives the downward pull of gravity as $f_g = 4/3\,\pi r^3\,(\rho_s - \rho_w)\,g$ where f_g is the force of gravity, ρ_s is the density of the sphere, and ρ_w is the density of the water. By equating f_g to f_v, the terminal velocity may be solved for, which is $V = 2\,r^2/9\mu\left[(\rho_s - \rho_w)g\right]$ and can be seen to be very small whenever the spheres are small.

Rapids are known to increase the formation of frazil ice. This is caused by the increased area of the water surface resulting from the agitation which causes increased cooling.

The amount of frazil ice formed as reported by Barnes[16] and Altberg[22] is 3 to 4 times as great as that which would be formed

on the surface as sheet ice. The average quantity of frazil ice formed in a winter will vary with the river and latitude. Barnes[16] gives an average value for the St. Lawrence River as 10 to 16 $ft.^3/ft.^2$ of exposed area and a maximum value for the Muira River in Canada as 23 $ft.^3/ft.^2$

Anchor ice is the ice formation which is found adhering to underwater objects. Barnes[16] indicates that anchor ice has been known to form at depths of 40 to 45 ft.

The process of cooling which causes anchor ice to form has been, and still is, a topic of considerable debate. Barnes contributes the cooling of underwater objects to nocturnal radiation. This theory was substantiated in his book[19] by the observation of many people who found that anchor ice only formed on cold clear nights and not under bridges or other cover. It was also noted to grow heavier on darker rocks. He further substantiated his theory by noting that anchor ice had been known to form below water which had a temperature which was measured and found to be slightly above the freezing point, thus ruling out the possibility of conduction of heat from the river bed or submerged objects as the cause of cooling, at least in these cases.

However, Altberg,[22] in refuting the theory of Barnes, claimed to have formed anchor ice under laboratory conditions excluding the possibility of radiation. In which case Barnes[16] claimed that Altberg had simply substituted refrigerant as the cooling agent.

Another objection to Barnes' theory which was raised by Gerdel[24] and by Gobletz[16] was the opaque nature of water to the radiation values beyond 2μ. The radiation of a body at the temperature of the earth in winter lies between 8μ and 20μ. Barnes[16] is able to over-rule this objection claiming that the infra-red spectrum of water shows an undoubted indication of a transparent band beyond 2μ. He further states that, "the ice balance is so delicate in water at 32^{o}F, and even more so in the case of supercooled water. Therefore a very small amount of energy, too small to be measured with a thermometer could be sufficient to cause the formation of ice, which when started would continue very rapidly."

Anchor ice has also been found by Schaffer[21] to be formed on fibrous material, which would tend to rule out radiation as a source of cooling since material of this nature has such a negligible mass that the radiational losses would not be sufficient to support an ice growth of the size shown in his illustrations. He, however, does not indicate the manner of cooling which permits it to form and says, "too little observational data and physical measurements are available to confirm or dispute the theory of Barnes that anchor ice forms due to the radiational cooling of the stream bed."

With the contradictory observations and measurements that have been made, one would have to agree with Schaffer that additional information is needed before the true nature of the

cause of anchor ice is ascertained. It is quite conceivable that both conduction and radiation cooling of the underwater objects are a cause of anchor ice depending upon the particular circumstances at the time of formation.

What is of most importance in this report is how the anchor ice affects the total volume of ice formed in the winter rather than what type of cooling causes it to form. Schaffer[21] describes anchor ice as being distinguished from frazil ice, as thin sheets which are firmly attached to underwater objects. The thickness to which they have been known to grow has exceeded 5 inches in one night.[16] In the sunlight anchor ice is often noted to break loose and rise to the surface often times carrying rocks and gravel. It is not known to form under an ice sheet although observations have been made indicating the contrary. Barnes[16] explanation for this is that the anchor ice forms prior to the ice cover.

The effect of anchor ice forming on the bottom is to raise the stage of the river. In areas where the banks are very low this could cause flooding. Where flooding is not a concern, the anchor ice of the bottom would have very little effect until it breaks loose in which case it can contribute significantly to the total volume of flowing drift ice or in building up an ice cover.

Thus far, the only ice cover which has been described is that formed as sheet ice on slowly moving water. Another type of ice cover is described by Stakle,[25] Stevens,[17] and Pariset and

Hausser[26] and is formed by the jamming together of flowing ice and can form with water velocities considerably higher than would permit a sheet of ice to form. For this type of ice cover both frazil and anchor ice are very important in contributing the majority of the ice that goes into the cover. A complete discussion and development of a theory for the prediction of the formation of an ice cover from moving ice is given in chapter 6. Therefore, only a brief description of some of the observations made and described in the literature will be made here.

Stakle[25] describes the formation of ice on the rivers of Latvia from surface drift ice. If a sufficient quantity of drift ice is formed during a prolonged cold spell, any natural or man made obstacle may cause it to jam together and cause an ice bridge. As the ice builds up behind the original jam, the added force of the ice may break it out or cause ice shoving and reestablishment of the equilibrium. If the force is not sufficient to cause a shove, a normal progression of the cover can occur. Either the ice shove causing jamming required to establish an equilibrium of forces within the cover or drift ice flowing under the cover and adhering to the underside of it can cause a partial blockage of the channel and backing up the water behind it. Such an occurrence is termed a hanging dam.[27]

Stevens[17] describes the hanging dam and calls it a "bridging gorge." He also describes what he calls a "flooding gorge."

Flooding gorges are known to form regularly on the Madison, Ruby, Boulder and sections of most other rivers and streams in Montana.

The requirements for the formation of a flooding gorge are a velocity of the river greater than will permit sheet ice to form, low banks, and sustained moderate temperatures between 15° and 25°F. The sustained moderate freezing temperature leaves the river open over most of its length which permits the formation of large quantities of frazil and anchor ice over a protracted period of time but not enough is manufactured at one time to form a bridging gorge.

The large quantities of frazil and anchor ice being carried by the water increase the apparent viscosity resulting in a decreased velocity of flow and increased stage. In areas where the banks are low and the river is broken up into a network of channels with many obstructions, the ice being carried by the water will become jammed and cause overflow of the river. This overflow carries with it large quantities of ice which are deposited in the low lying areas to freeze. Also contributing to the rapid freezing and build-up of the new and old channels with ice, is the further decrease in velocity due to the jamming and the coming into contact of the ice laden water with the new channel beds and low lying areas which did not have the prior insulating effect of the water, resulting in rapid freezing together of the drift ice. Continuation of the cold spell results in a continued build-up and flooding of the river valley as the river searches and finds new

channels. The depth of ice may reach several feet over the entire valley floor forcing a complete evacuation of the area.

Most rivers in Montana will form all three types of ice described as well as the various types of ice covers and gorges. It is primarily a natural occurrence and, although the processes can be altered by man, it is not likely that a bridge will be the total cause of ice problems in an area. A bridge can contribute to increasing a problem which already exists as was described in earlier chapters.

CHAPTER 6

ICE COVER FORMATION FROM DRIFT ICE

When the velocity of the surface of the water exceeds the velocity at which a smooth ice sheet will form, the ice cover must be made up of floating drift ice which has been jammed together. The formation of the initial ice bridge behind which the ice cover evolves may be aided by either natural or man made obstacles. Natural obstacles could include a narrowing of the river, sharp curves, shoals, abrupt change in slope, or a natural ice sheet formed over a more placid stretch of the river. The man made obstacles could include bridges, low diversion dams, control dams, road fill, etc.

The drift ice making up the ice cover can be composed of shell, anchor, shore, and slush ice. Shell ice is frazil ice which has united into thin round sheets. Anchor ice, which has broken away from the bottom, and shore ice, which has broken away from the shore, mix in with the shell ice and add to the total volume of ice being carried by the surface of the water as drift ice. Also, precipitation in the form of snow falling into the river can form into slush ice which can contribute significantly to the volume of the drift ice.

If the drift ice becomes sufficiently thick so that it covers almost the entire water surface, any natural or man made obstacle may cause it to join together and form into an ice bridge across

the entire river.[17,25,26,27] An ice cover may then extend upstream from the ice bridge by further accumulation of drift ice.

The extent of ice cover and the thickness to which it will develop is a function of the water velocity and amount of ice available for its formation.

Pariset and Hausser[26] have developed mathematical expressions as a function of the water velocity which supposedly predict the rate, type, and extent of ice cover build-up resulting from drift ice. Morton,[27] in a later paper, modified these equations to take into account the effect of the difference in roughness between the river bed and bottom of the ice cover.

The development of these equations was based on the following assumptions:

1. Straight rectangular channel.
2. Uniform velocities.
3. Width of channel much greater than the depth.
4. The ice floes to be cohesionless and have an angle of internal friction equal to zero.

Presented herein are the more significant equations which were developed along with some explanation as to their usage.

Once the ice bridge has been formed either by a natural or man made obstruction jamming the ice together, the normal thickness of the continued build-up of the cover can be predicted using equation 6-1.[26]

$$V = \sqrt{2g \frac{\rho - \rho'}{\rho} t \left(1 - \frac{t}{H}\right)} \qquad (6\text{-}1)$$

where: (See figure 6-1)

V is the velocity of the water in front of the cover in Ft./sec.

g is the acceleration of gravity in Ft./sec.^{2}

ρ is the specific gravity of water, equal to 1.

ρ' is the specific gravity of ice.

t is the normal thickness of ice cover build-up in ft.

H is the depth of flow of the open channel in ft.

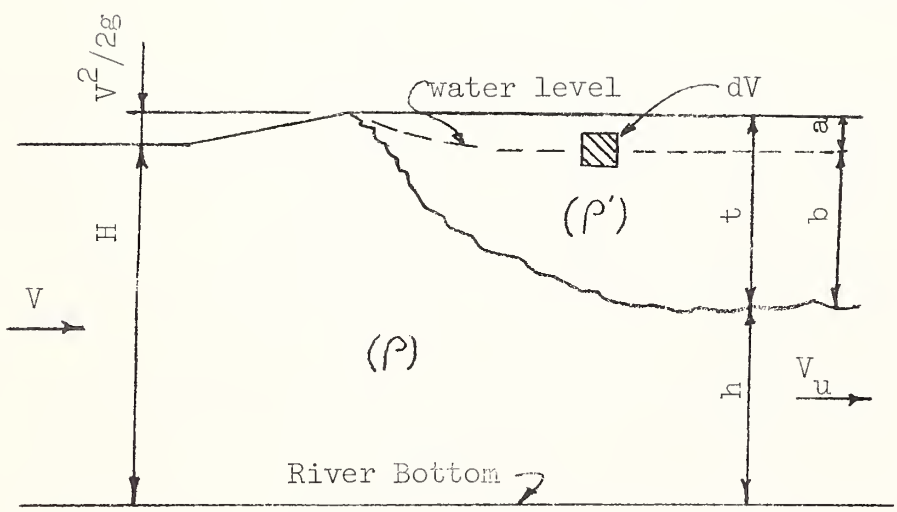


Figure 6-1

Channel Section at Upper End of Ice Cover

Equation 6-1 gives two values of t for each value of V. However, only that root of the equation which gives the smallest positive value of t is significant.

The critical value for the velocity can be obtained by rewriting equation 6-1 in the form given as equation 6-2 and

differentiating the velocity with respect to the relative thickness of ice cover t/H.

$$\frac{V}{\sqrt{2gH}} = \sqrt{\frac{\rho - \rho'}{\rho} \frac{t}{H} (1 - t/H)} \tag{6-2}$$

Setting the differentiated equation of Equation 6-2 equal to zero will give a relative thickness, t/H, of ice cover equal to 0.33 which when substituted into equation 6-2 along with the specific gravity of ice, ρ' equal to 0.92, results in the equation for critical velocity given by equation 6-3.[26]

$$V_c = 0.109 \sqrt{2\ gH} \tag{6-3}$$

A velocity greater than that given by equation 6-3 will cause submergence of the incoming ice floes and prevent any further progression of the ice cover.

The normal progression of the ice cover may also be stopped if the thickness, d, of the ice floes making up the drift ice is less than that given by equation 6-4.[26]

$$d_c = \frac{V^2}{2g\ K^2 \left(\frac{\rho - \rho'}{\rho} \right)} \tag{6-4}$$

where:

d_c is the critical thickness of ice floes in ft.

K is the form factor which varies from 0.66 for cubic floes to 1.3 for thin floes.

In the case of either the critical velocity being exceeded or the thickness of the ice flows being less than the critical

thickness, the edge of the cover may not be able to obtain its minimum thickness and continue to build up since all of the incoming ice flows are possibly being submerged and carried under the cover.

The transportation of ice floes under the cover was found by tests[26] to be similar to the transportation of river beds. Thus the Meyer-Peter's equation may be used and, assuming the hydraulic radius is equal to half the mean depth, h,* it becomes:

$$1000 \frac{V_u^2}{C^2} = 3.75\, d + 5\, q_{si}^{2/3} \qquad (6\text{-}5)$$

where:

V_u is the average velocity under the ice cover in Ft./sec.

C is the Chezy roughness coefficient of the cover.

q_{si} is the discharge of ice from under the cover in #/ft. of width weighted under water with an apparent specific gravity of 0.08.

Further build-up of the cover will not occur if the critical velocity is exceeded or if the incoming ice has a sub-critical thickness as long as the amount of incoming ice is less than that which can be transported under the cover. Thus, a condition of equilibrium is reached and only if this equilibrium is disturbed will the cover continue to build.

---

* When the ratio of the width, B, to mean depth, h, is greater than 50, the error in this approximation is less than 2%.

The disturbing of this equilibrium can be the result of a reduction in the velocity of the water, an increase in thickness of the ice floes, or an increase in the amount of incoming ice.

It is obvious that if the build-up of the cover ceased because the ice floes were sub-critical in thickness, an increase in thickness to the critical thickness or greater would permit continued build-up. Also, if more ice comes into the head of the cover than can be transported under it, that which cannot be transported must contribute to the continued cover build-up.

The reduction in the velocity of the water can be the result of causes which are not directly related to the ice cover itself. Such causes could be a reduction in flow from a control or hydro-electric facility, a large quantity of ice being manufactured and held upstream, or a reduction in flow from the natural sources caused by freezing of the springs or tributaries.[28] If the reduction in velocity is such that the incoming floes cease to be submerged, the natural build-up of the cover will of course continue.

The reduction of the velocity at the upper edge of the cover may, however, be a result of a reduced flow under the cover as a result of its thickening. Some of the ice which is submerged and carried under the cover is likely to come into contact with the under surface of the cover and adhere to it. Also, a water velocity in excess of the critical velocity for the normal thickening

of the cover as given by equation 6-1 can cause a thickening of the cover by shove. The assumption in the development of these equations was that the ice is in a cohesionless state at this stage of the ice cover evolution. Therefore, if the thrust at any point in the cover becomes greater than the resisting thrust, plastic flow must occur to establish equilibrium resulting in a thickening of the cover. This thickening of the cover may reduce the flow of water under it and in so doing raise the stage of the river behind it, thus reducing the velocity and permitting a continued build-up of the cover.

In order to be able to predict the phenomenon of shove, it is necessary to investigate the forces applied to the cover as well as the resisting forces.

The force tending to move the cover is the sum of forces from four sources of loading. These sources are:

1. Thrust on the upstream edge of the cover. (f_o in #/ft. of width)
2. Drag of water on the under surface of the ice. (f_d in #/ft.^{2} of surface area)
3. Drag of wind on the upper surface of the ice. (f_w in #/ft.^{2} of surface area)
4. Gravity component of the ice. (f_g in #/ft.^{2} of surface area).

The resisting forces are:

1. Cohesion and friction of the banks.

2. Internal strength of the ice cover.

The thrust on the upstream edge of the cover can easily be determined by utilizing the assumptions that the ice is cohesionless and has a coefficient of friction equal to zero. For such a cohesionless material, the pressure at the water line resulting from the weight of the ice above must be equal to the buoyant upward force of the displaced water below. (See figure 6-1) Such a situation will result in a stress distribution from top to bottom of the ice cover as shown in figure 6-2 with the maximum stress occurring at the water level.

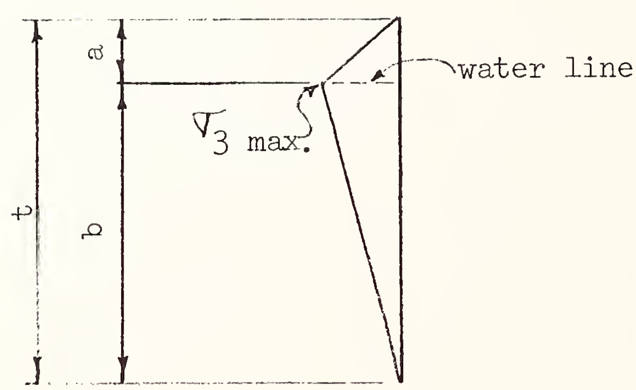


Figure 6-2

Stress Distribution in the Ice Cover

Taking out a differential volume, dV, at the water level the stresses acting on it would be as shown in figure 6-3. However, assuming that the angle of internal friction between the various ice floes making up the ice cover is zero: $\sigma_1 = \sigma_2 = \sigma_3$

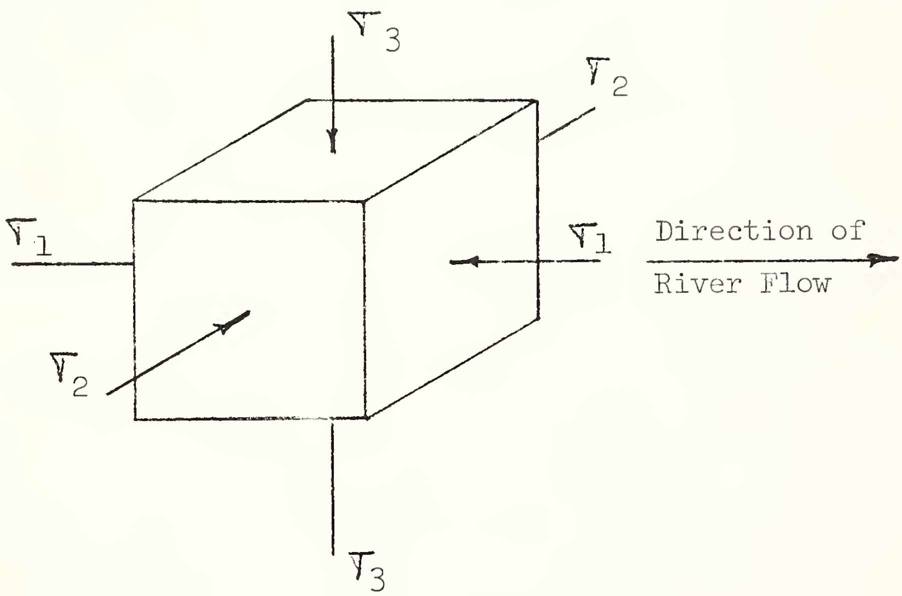


Figure 6-3

Differential Stress Block

$\sigma_{1\ max.}$ at the water line is then equal to $\sigma_{3\ max.}$ which in equation form becomes:

$$\sigma_{1\ max.} = \sigma_{3\ max.} = \gamma_i a$$

where:

$\sigma_{1\ max.}$ is the stress on the edge of the cover at the water level in. $\#/ft.^2$

γ_i is the unit weight of ice in $\#/ft.^3$

a is the distance from the top of the cover to the water level in ft.

The force per foot of width, f_o, acting on the front of the cover is obtained by summing the stress over the entire thickness. (See figure 6-2).

$$f_o = \frac{\gamma_i a\ t}{2}$$

However,

$$t = a + b$$

and the buoyant condition of the ice gives the equation:

$$t = \frac{\rho}{\rho'}\ b$$

where b is the distance from the water surface to the bottom in ft.

Using the last two equations to solve for a in terms of t and substituting this value of a into the equation for the thrust on the front of the cover results in equation 6-6.

$$f_o = \frac{\gamma_i t^2}{2} \left(1 - \frac{\rho'}{\rho}\right) \qquad (6\text{-}6)$$

The thickness of the ice cover, t, can be eliminated from equation 6-6 by substituting in the value of t from equation 6-7.[26]

$$V_u = \sqrt{2g\ \frac{\rho - \rho'}{\rho}\ t} \qquad (6\text{-}7)$$

where $V_u \doteq V/(1-t/H)$ and is the average velocity beneath the ice cover in ft./sec. Equation 6-6 then becomes equation 6-7.

$$f_o = \frac{1}{8}\gamma_i \frac{\rho}{\rho - \rho'} \frac{V_u^4}{g^2} \qquad (6\text{-}7)$$

The drag force on the bottom of the ice resulting from the flow of water can be developed by considering a short reach of river with a uniform cross section, A, and length L. (See figure

6-4). Although there is water above the bottom of the ice cover, it is assumed that none is flowing through it.

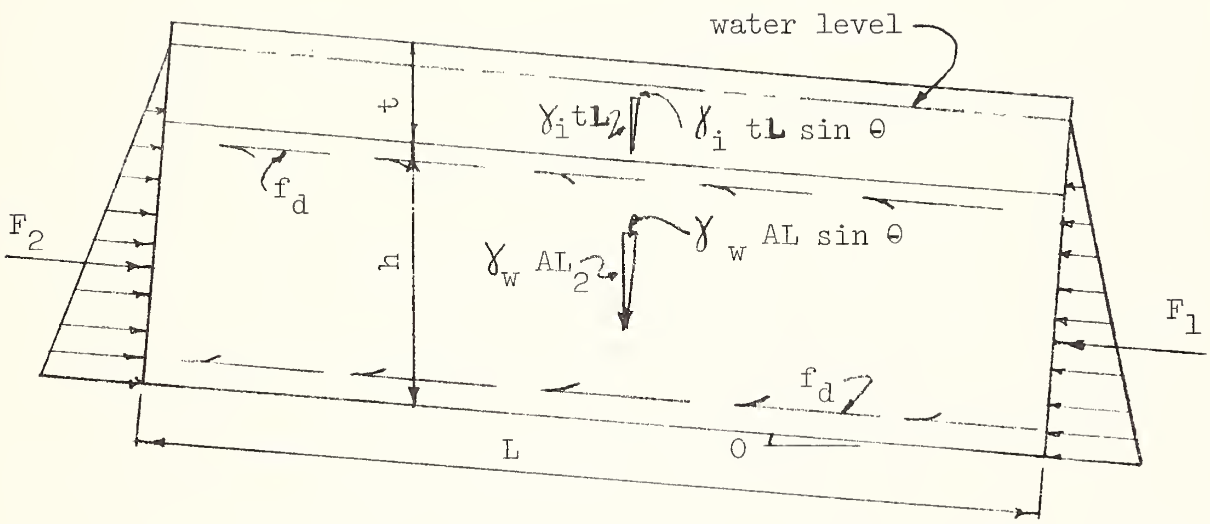


Figure 6-4

Typical Channel Section Under Ice Cover

If it is assumed that there is uniform flow through this section, the body of water contained can be assumed to be in static equilibrium. Summing the forces in the direction of flow gives:

$$\Sigma F = 0 = F_1 + \gamma_w AL \text{ Sin } \theta - f_d PL - F_2$$

where γ_w is the unit weight of water in #/ft.^{3} and P is the wetted perimeter in ft. If h is a constant, then $F_1 = F_2$ and

$$f_d = \frac{\gamma_w AL \text{ Sin } \theta}{PL} = \gamma_w R \text{ Sin } \theta.$$

where R is the hydraulic radius in Ft. However, Sin θ = S the slope of the river and

$$f_d = \gamma_w R\ S \qquad (6\text{-}8)$$

It is possible to write equation 6-8 in terms of the velocity of flow under the cover, V_u, by substituting into it the value of S obtained from the Chezy equation ($V_u = C\sqrt{RS}$).[29] Equation 6-8 then becomes equation 6-9.[25]

$$f_d = \frac{\gamma_w V_u^2}{C^2} \qquad (6\text{-}9)$$

where C is the coefficient of roughness for the Chezy Equation. Equation 6-8 may also be written in terms of V_u by substituting into it the value of S obtained from the Manning equation ($V_u = \frac{1.49}{n} R^{2/3} S^{1/2}$).[29] The result is equation 6-10.[27]

$$f_d = \frac{\gamma_w n^2 V_u^2}{(1.49)^2\ r^{1/3}} \qquad (6\text{-}10)$$

where n is the coefficient of roughness for the Manning Equation. For both equations 6-9 and 6-10, the coefficient of roughness n or C was assumed to be the same for both the ice and the river bed. Morton[27] states that the Manning roughness coefficient for the ice could conceivably vary from a high of 0.06 to a low of 0.012 during the complete evolution of an ice cover, while the roughness coefficient for the river bed he gave as having an average constant value of 0.03. It is expected that the ice is quite rough during the early formation but it is smoothed out by the heat from the friction of the water and the accretion of underwater ice.

To take into account the difference in the roughness of the

river bed and ice cover, Morton[27] developed a relationship from which can be calculated an effective Manning roughness coefficient that can be used in equation 6-10. This relationship is given as equation 6-11.

$$n_e = n_i \left[\frac{1 + \dfrac{n_b}{n_i}^{3/2}}{2} \right]^{2/3} \qquad (6\text{-}11)$$

where:

n_e is the effective Manning coefficient of roughness.

n_b is the Manning coefficient of roughness for the river bed.

n_i is the Manning coefficient of roughness for the bottom of the ice cover.

Comparing the values of n_e using equation 6-11 for various values of n_i and n_b to the average value of n_i and n_b for each case, indicated a very close correlation. Thus, it is felt that, with the great uncertainty in the correctness of the values of both n_b and n_i used, the simpler method of taking the average of the two values to obtain the effective Manning roughness coefficient (equation 6-12) would be sufficient for the use in these equations.

$$n_e = \frac{n_i + n_b}{2} \qquad (6\text{-}12)$$

An effective Chezy coefficient for roughness as an average of the Chezy roughness coefficient for the ice and the Chezy roughness coefficient for the river bed (equation 6-13) may also

be used when using equation 6-9 to take into account the differences in roughness of the two surfaces.

$$C_e = \frac{C_i + C_b}{2} \qquad (6\text{-}13)$$

where:

C_e is the effective Chezy coefficient for roughness.

C_i is the Chezy coefficient for roughness of ice.

C_b is the Chezy coefficient for roughness of the river bed.

The variation in the Chezy roughness coefficient to correspond to the variation in the Manning coefficient for roughnesses from 0.012 to 0.06 for a particular river cross section with R equal to 15 is from 195 to 40. The average value for the river bed would be around 80.

The justification for the use of open channel theory in developing these equations rests with the assumption of a cohesionless ice cover during the period of formation. Essentially all that has been assumed to happen is that the air cover has been replaced by an ice cover and until the cover solidifies no significant internal pressure can be developed. However, Kennedy[30] found that a form of the Karman-Prandtl[31] equation for velocity distribution near rough boundaries could be used to calculate the friction drag forces under a pulpwood jam.

Since there is a considerable similarity between a pulpwood jam and an ice cover, it is felt that such an equation might also

be used in the calculation of the friction drag for the ice cover. The equation is given as equation 6-14.

$$f_d = \frac{\gamma_w \, V_y^2}{g\,(5.75 \log y/k_e + 8.5)^2} \qquad (6\text{-}14)$$

where:

V_y is the velocity at a distance y feet below the mean undersurface of the jam in ft./sec.

k_e is the mean value of k, the equivalent roughness of the under surface of the ice cover at a point in ft.

If the average velocity V_u is assumed to be at a distance y from the mean undersurface of the ice cover, (See figure 6-5), the equation can be rewritten as equation 6-15.

$$f_d = \frac{\gamma_w \, V_u^2}{g\,(5.75 \log y/k_e + 8.5)^2} \qquad (6\text{-}15)$$

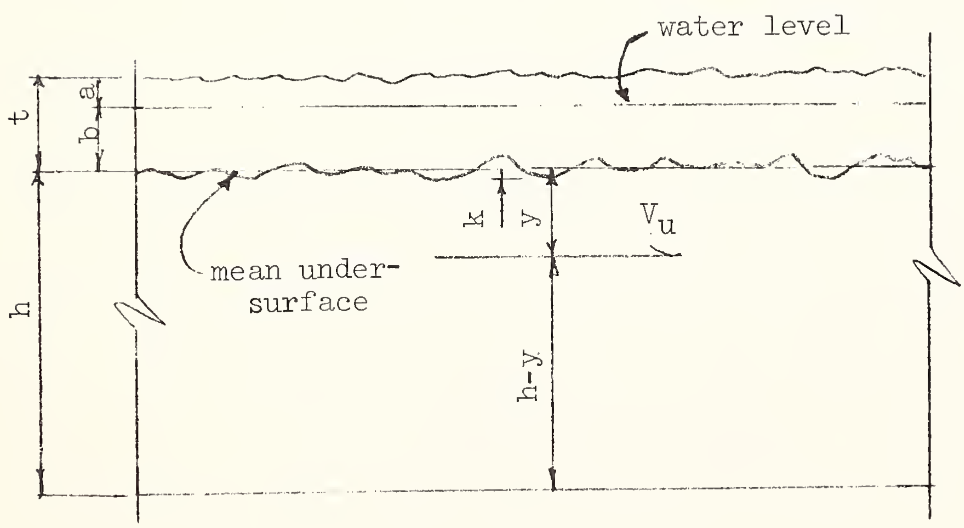


Figure 6-5

Typical Channel Section Under Ice Cover

Before equation 6-15 can be used, some value for the equivalent roughness, k_e, must be determined as well as the distance, y, to a point where the velocity can be assumed to be equal to the average value V_u. Thus, the equation at this time can only be presented as a possible alternative to the use of equations 6-9 and 6-10 until additional field and laboratory data is available on the roughness and water flow characteristics under an ice cover.

The determination and use of a proper roughness coefficient for the ice cover is further complicated by the fact that it will change with time. Therefore not only the knowledge of the roughness of the undersurface of the ice cover is necessary but also needed is some knowledge of the change of roughness with respect to time.

The gravity component of the ice looking at figure 6-4 is given by equation 6-16.

$$f_g = \gamma_i t \operatorname{Sin} \Theta = \gamma_i t S \qquad (6\text{-}16)$$

However, S can be eliminated by substituting for it the value obtained from the Chezy equation resulting in equation 6-17.

$$f_g = \frac{\gamma_i t V_u^2}{R C_e^2} \qquad (6\text{-}17)$$

Equation 6-16 may also be written in terms of V_u by substituting for S the value obtained from the Manning equation which results in equation 6-18.

$$f_g = \frac{n_e^2 \, \gamma \, i \, t \, V_u^2}{(1.49)^2 \, r^{4/3}} \qquad (6\text{-}18)$$

The same discussion as applied to the roughness coeffieients for equations 6-9 and 6-10 is also applicable to the roughness coefficients in equations 6-17 and 6-18.

Both Morton[27] and Pariset and Hausser[26], although indicating that there could be a wind force, proceeded to neglect it in the final analysis as being insignificant. However, in the analysis of forces for pulpwood holding grounds,[30] wind forces were taken into account and calculations indicate that in this case they were quite significant. For a wind with a velocity of 30 MPH at a height of 6 feet above the pulpwood jam, the forces calculated were as much as 25% of the forces calculated for the total water thrust.

The relationship used to calculate the wind force is a form of the Karman-Prantdl equation and given as equation 6-19.[30]

$$f_w = \frac{\gamma_a \, V_y^2}{g \, (5.75 \text{ Log } y/k_e + 8.5)^2} \qquad (6\text{-}19)$$

where γ_a is the unit weight of air in #/ft.^{3}

As can be seen by inspection of equation 6-19, the magnitude of the wind force is not only a function of the wind velocity but is also dependent upon the effective roughness of the surface over which the wind passes. If this effective roughness is very small, then it can be expected that the wind force would also be quite small in which case the neglecting of the force in the calculations may be permissible. Until, however, it can be shown, by test, to be insignificant, no development can be assumed to be complete without including it.

The resisting force of the banks is made up of two parts, the cohesion and friction. Therefore the equation for the resisting force of the banks may be written as equation 6-20.

$$R_b = T + \mu P \qquad (6\text{-}20)$$

where:

R_b is the resisting force of the banks, in #/ft.

T is the cohesion force, in #/ft.

μ is the coefficient of friction between the bank and ice.

P is the lateral pressure of the ice against the banks, in #/ft.

The internal strength of the cover can be calculated in the same manner as the thrust on the front of the cover, equation 6-6, as long as it is permissible to assume a cohesionless state for the cover.

$$R_i = \frac{\gamma_i \, t^2}{2} \left(1 - \frac{\rho'}{\rho}\right) \qquad (6\text{-}6)$$

where R_i is the resisting force of the ice, in #/ft. of width.

The equilibrium of the system can now be determined by equating the acting forces to the resisting forces. The free-body of a portion of the cover is shown in figure 6-6.

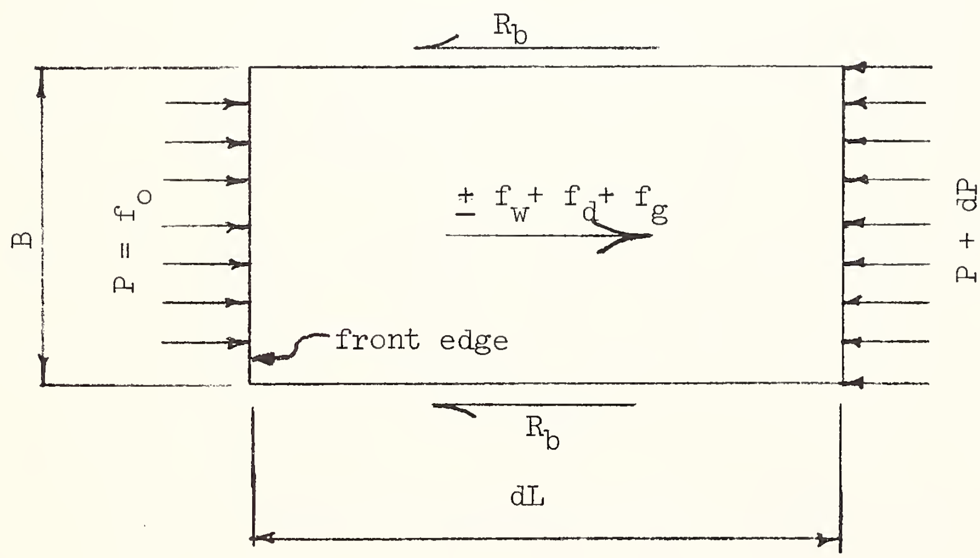


Figure 6-6

Freebody of a Section of Ice Cover

From the freebody (fig. 6-6) the differential equation for equilibrium at any point may be written.

$$\Sigma F = 0 = P\,B + (f_d + f_g \pm f_w)\,BdL - 2\,R_b\,dL - (P + dP)B$$

$$\therefore \frac{dP}{dL} + \frac{2\,R_b}{B} = (f_d + f_g \pm f_w)$$

However, the friction force included with the resisting force of the bank is a function of P and must be included in the equation before it can be solved. The final differential equation is then given as equation 6-21.

$$\frac{dP}{dL} + \frac{2\mu P}{B} = f_d + f_g \pm f_w - \frac{2T}{B} \qquad (6\text{-}21)$$

Integrating this equation and solving for the constant of integration by using the boundary condition at the front edge; that is, when L = 0, $P = f_o$, the final equation becomes equation 6-22.[26]

$$P_L = (f_d + f_g \pm f_w)\frac{B}{2\mu} - \frac{T}{\mu} - \left[(f_d + f_g \pm f_w)\frac{B}{2\mu} - \frac{T}{\mu} - f_o\right]e^{\frac{-2\mu l}{B}} \qquad (6\text{-}22)$$

where:

P_L is the thrust on the ice cover at a distance L from the upstream edge of the ice cover in #/ft.

B is the width of the river in ft.

However, the equilibrium of the cover requires that $P_L = R_i$. Therefore, referring to equation 6-22, if f_o is larger than the value $\left[(f_d + f_g \pm f_w)\frac{B}{2\mu} - \frac{T}{\mu}\right]$, the river is termed a narrow river in

which case the internal force will always be less or at most equal to the thrust on the upstream edge of the cover. For such a cover, there will be no thickening of it by shove and, knowing the velocity of the water, depth of channel, and specific gravity of the ice, the thickness can be calculated using either equation 6-1 or equation 6-7 after which the maximum thrust can be calculated using equation 6-6.

If on the other hand f_o is less than the value $[(f_d + f_g \pm f_w) \frac{B}{2\mu} - \frac{T}{\mu}]$, the river is termed wide whereby the thrust increases with the distance from the front edge of the cover. Thus thickening of the cover by shove must occur if internal equilibrium is to be maintained. This thickening of the cover may or may not cause damming of the river and reduction in the velocity of flow as will be shown later in the report.

Theoretically, for the case of a wide cover, the maximum thrust would occur when $e^{-2\mu L/B}$ goes to zero, which is the case when L approaches infinity. The resulting equation would then be equation 6-23.

$$P_L = (f_d + f_g \pm f_w) \frac{B}{2\mu} - \frac{T}{\mu} \tag{6-23}$$

For all practical purposes it can be said that $e^{-2\mu L/B}$ goes to zero when $L > 2B$. Therefore, equation 6-23 could be used to calculate the thrust at any point beyond the distance 2B from the front edge of the cover. However, upon substituting f_d, f_g, and f_w

into equation 6-23 (equation 6-24), it is found that t is as yet unknown thus making it impossible to solve for the thrust.

$$P_L = \frac{B}{2\mu} \frac{V_u^2}{C_e^2} \left(\gamma_w + \frac{\gamma_i t}{R} \right) \pm \frac{\gamma_a V_y^2}{g\,(5.75 \log y/k_e + 8.5)^2} - \frac{T}{\mu} \qquad (6\text{-}24)$$

By utilizing the relationship $P_L = R_i$ it is possible to equate equations 6-23 and 6-6 and then, substituting in $V/(1 - t/H)$ for V_u and $\frac{H - t}{2}$ for R and knowing the other quantities in the equation, it is possible to solve for t. The resulting equation is given as equation 6-25.

$$\gamma_i \left(1 - \frac{\rho'}{\rho}\right) \frac{t^2}{2} = \frac{B}{2\mu} \frac{V^2}{C_e\,(1 - t/H)^2} \times \left(\gamma_w + \frac{2\gamma_i t}{H - t} \right) + \frac{\gamma_a V_y^2}{g\,(5.75 \log y/k_e + 8.5)^2} - \frac{T}{\mu} \qquad (6\text{-}25)$$

Once t is calculated using equation 6-25, it can be substituted into equation 6-6 to determine the maximum thrust at the point in question.

Equation 6-24 and 6-25 could have also been written using the equations for f_d and f_g which used the Manning Equation in the development. The results would be essentially the same.

Before equation 6-25 can be used, the values for T and μ must be known or determined. These values were derived by Pariset and Hausser[26] for the St. Lawrence River and Beauharnois Canal. The way in which they arrived at their values was to rearrange equation 6-24, neglecting the wind force, into equation 6-26. Then, ignoring

T and μ, they plotted experimental points using $\frac{BV_u^2}{CR}(R + \frac{\rho'}{\rho}t)$ as the ordinate and $\frac{\rho'}{\rho}(1 - \frac{\rho'}{\rho})t^2/2$ as the abscissa. These plots showed that the experimental points were scattered along a straight line which must represent the plot of equation 6-26. Thus it was possible to obtain μ and T as constants to satisfy equation 6-26.

$$\frac{B\ V_u^2}{2\mu C^2 R}\left(R + \frac{\rho'}{\rho}t\right) = \frac{T}{\gamma_w \mu} + \frac{\rho'}{\rho}\left(1 - \frac{\rho'}{\rho}\right)\frac{t^2}{2} \qquad (6\text{-}26)$$

The value of bank cohesion T for the St. Lawrence River was given as 75 #/ft. and for the Beauharnois Canal was given as 90.5 #/ft., the coefficient of friction was given as 1.28. The value of T was found to be independent of the thickness of the ice cover. The reason given by Pariset and Hausser[26] for the constant value of cohesion was that the thick ice covers were more or less broken thus reducing the cohesion.

There appears to be some question as to the validity of assuming any bank cohesion at all since the assumption of a cohesionless cover was made for the original development of the theory. It may, however, be justified in that the velocity along the edges of the river are generally low enough, at least in many areas, for the formation of a smooth ice sheet adhering to the banks and extending out toward the center of the river. This shore ice makes up just a small portion of the total ice cover and as such is not too significant in the general development of the theory with the exception of

adding to the resistance to shoving of the ice cover. When the ice breaks up in the spring it is expected that the value of bank cohesion goes to zero.

Actually there is some doubt as to whether or not the values of T and μ, so obtained, really represent the true values. It is felt, by the writers, that the manner in which these values were obtained indicate that they only represent constants to take into account the general resistance to shove which could also include added internal resistance caused by consolidation and freezing together of the ice floes with time as well as any bank cohesion and friction that might be developed. It might also be found that the assumption that the angle of internal friction is zero is incorrect. In which case an investigation to determine this value would be necessary.

The general development of the equations was completely independent of time or the amount of ice available for the building of the cover. It has been assumed that the cover remains in a cohesionless state indefinitely or at least until the equilibrium condition is reached with the exception of adhering to the banks. A more realistic approach might be to account for some of the consolidation and freezing of the ice floes as the cover progresses thus increasing the internal resistance over that which is predicted by the general theory.

A look at the rate of cover build-up would show that there is sufficient time for a change in the internal resistance to occur from the additional freezing of the cover. The rate of build-up of an ice cover is a function of the percentage of water covered by the drift ice, mean thickness of the drift ice, velocity of water flow, and channel properties. For a 4000 ft. wide river, 30 ft. deep, with a Chezy coefficient of roughness equal to 60 and water velocity of 2 ft./sec., Pariset and Hausser[26] found, that when 20% of the water surface was covered with drifting ice with a mean thickness of 0.2 ft., the cover would progress at a rate of 70 ft./hr. which they considered to be a very low rate. With 80% of the water surface covered by drift ice with a mean thickness of 0.2 ft., the rate of progression went up to 340 ft./hr. Theoretically even much higher rates of progression could be expected. Equation 6-23 when used to determine the thrust at a section, or equation 6-25 used to establish equilibrium of a wide cover, is only valid at a distance from the upstream edge of the cover equal to 2B or more. For the 4000 ft. wide channel this would be 8000 ft. from the upstream edge. Assuming the rate of cover build-up to be 500 ft./hr., it would take 16 hrs. for the cover to progress this 8000 ft. which appears to be sufficient time for the drift ice, which is jammed together and essentially motionless, to freeze together and increase the internal resistance to shove.

It might be found that, if there is an increase in the internal resistance caused by inter-freezing of the drift ice, it can be considered in the present development to be independent of time. Both the rate of freezing together of the drift ice making up the cover and the rate of manufacturing of the drift ice to extend the cover depend upon the amount of heat transferred from the water. Thus, if it is moderately cold, the manufacturing of drift ice and the build-up of the ice cover would proceed slowly and, at the same time, the freezing together of the drift ice making up the cover would also be expected to proceed at a slow rate. Conversely, if it was very cold, both the build-up of the cover and inter-freezing of the drift ice making up the cover would tend to proceed more rapidly. Therefore, if the rate of increase in the internal resistance to shove by the freezing together of the drift ice can be shown to be directly proportional to the rate of cover build-up, equation 6-22 may perhaps be modified to take into account this increase in internal resistance independent of both time and shore resistance.

It should further be noted that equation 6-22 is approximate in that both f_d and f_g which, in the equation, are assumed to be constant actually vary as some function of the length. The error in this assumption is not felt to be significant, particularly as the length increases. However, some

investigation into the accuracy should perhaps be made if the equation is to be used for the prediction of ice cover equilibrium close to the upstream edge.

In general it is felt that, because of the manner in which the values for T and μ were obtained for equation 6-22 and the lack of general verification of the theory, the equation can at this time only be assumed to be an empirical relationship for the prediction of ice cover development on the St. Lawrence River and Beauharnois Canal. The relationships so developed do, however, give a starting place and with the completion of considerable work yet necessary to verify or modify the equations a general theory could eventually be developed.

To begin with, particular attention must be given to the obtaining of additional data on the shore resistance as well as the possible variation in the internal resistance of the ice cover. This would require an investigation which could separate the various factors involved in the resistance of an ice cover to shove and check them independently under all conditions of ice cover development.

The point of transition between a narrow and wide ice cover can best be shown by plotting equations 6-1 and 6-25. Also, from such curves, if the theory is at all valid, it is possible to predict the type of ice cover which is likely to form knowing the velocity of the water.

From equations 6-1 and 6-25 a plot of the velocity ahead of the ice cover versus the thickness of ice can be made for a particular river cross-section. Such a plot is shown in figure 6-7 for a river cross-section 1200 ft. wide, 15 ft. deep and assuming ρ' equal to 0.92, C_e equal to 75, μ equal 1.28, and T equal to either 0 or 75 #/ft. This cross-section could perhaps represent the Yellowstone River down around Glendive, Montana.

Curve 1 of figure 6-7 represents the normal thickness of ice for a narrow cover or front edge of a wide cover obtained by plotting equation 6-1. Curve 2 represents the thickening of the ice using equation 6-25 and assuming a wind of 30 MPH, 6 ft. above the ice in the direction of the river flow and a coefficient of roughness, k_e, for the ice of 1.5. Curve 3 is the same as curve 2 but includes no wind. Curve 4 represents the thickening of the ice cover using equation 6-25 but includes no cohesion or wind. Curve 5 is the same as curve 4 but includes the wind similar to curve 2.

The transition from a narrow to a wide cover occurs when the curve plotted from equation 6-25 meets the curve plotted from equation 6-1. For the case of cohesion but no wind, curve 3, the cover of the 1200 ft. wide river would thicken in a normal manner as a narrow cover as long as the velocity of flow did not exceed 2.75 ft./sec. However, if the velocity does exceed this value a

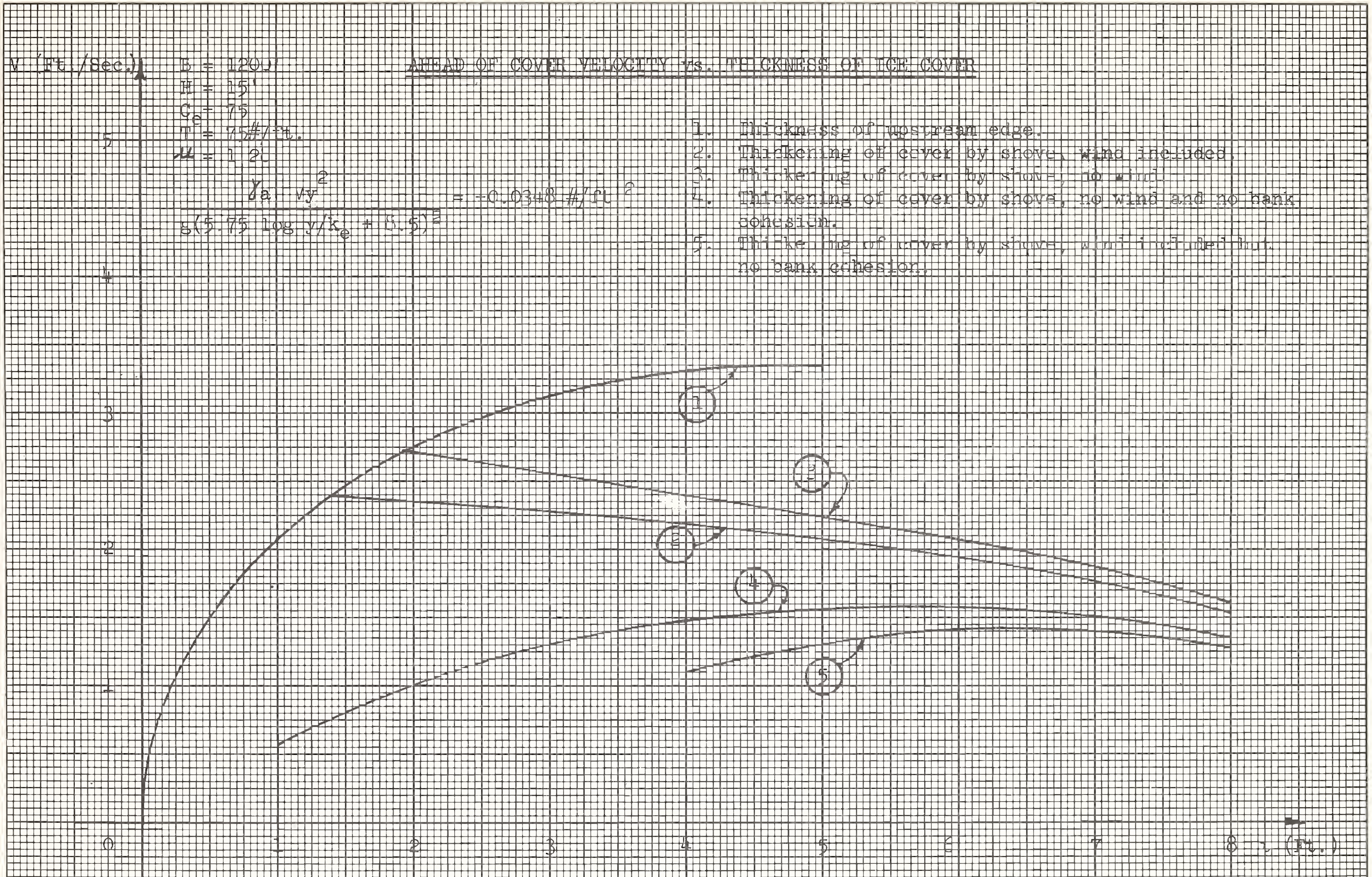

AHEAD OF COVER VELOCITY Vs. THICKNESS OF ICE COVER
V (Ft./Sec.)
B = 1200'
H = 15'
C_e = 75
T = 75#/ft.
μ = 1.20
$\dfrac{\gamma_a\ V_y^2}{g(5.75 \log y/k_e + 8.5)^2} = +0.0348\ \#/ft^2$
1. Thickness of upstream edge.
2. Thickening of cover by shove, wind included.
3. Thickening of cover by shove, no wind.
4. Thickening of cover by shove, no wind and no bank cohesion.
5. Thickening of cover by shove, wind included but no bank cohesion.
1
2
3
4
5
0
1
2
3
4
5
6
7
8
t (Ft.)

FIGURE 6-7

very unstable situation is indicated by the downward slope of curve 3 which indicates, theoretically at least, that the cover could thicken down to the bottom of the stream. Actually what generally will happen is the cover will thicken by shove and cause damming of the water which results in a reduction in the water velocity sufficient to permit equilibrium of the cover and continued build-up. This process may continue as long as there is sufficient drift ice to feed the front edge of the cover. With time the cover will solidify sufficiently to become stable and the friction of the water will eventually smooth the under surface so that full normal flow is permitted. This is what is most often referred to as a hanging dam.[17,27] For this type of situation the total thickness of the ice cover would be very difficult to determine using the present theory alone.

The amount of jamming and rise in water level depends upon the particular river, velocity of the water and the amount of ice available to go into the formation of the cover. In some cases the water may rise sufficiently to cause overflow of the banks and flooding of the surrounding areas before equilibrium is reached. Such a condition of flooding is a yearly occurrence on some rivers which have low banks. This overflow generally permits a sufficient decrease in the velocity of the water to permit equilibrium of the cover. However, the overflow water is carrying with it large quantities of ice which will be deposited in the low lying areas

resulting at times in a large area being built up with ice. This phenomenon is referred to as ice gorging.[17] Eventually the water in the main channel will either smooth out the undersurface of the cover sufficiently to permit full flow of the river or possibly wash out a channel through the cover leaving thick deposits of the cover extending out from each bank. For this situation it would again be very difficult to predict, at the present time, the total thickness of the ice build-up.

Anytime during this period of instability, it is possible that the original ice jam is washed out leaving the entire river open. This would certainly be the case if the velocity of flow was never sufficiently decreased by damming to permit a condition of equilibrium to develop.

Looking at curve 4, the case of no bank cohesion and no wind, the thickening of the ice cover by shove occurs almost immediately and as long as the velocity does not exceed 1.65 ft./sec. an equilibrium condition will develop without jamming. If the velocity is 1.65 ft./sec. the depth of the cover to be in equilibrium is already $5\frac{1}{2}$ ft. Additional velocity would cause even greater thickness and damming of the river resulting in the same unstable situation that was discussed previously.

A comparison of curves 3 and 4 or 2 and 5 shows how sensitive the point of transition between a narrow and wide cover is to changes in the cohesion. This is better shown on figure 6-8 where

EFFECT OF T ON THICKENING VELOCITY

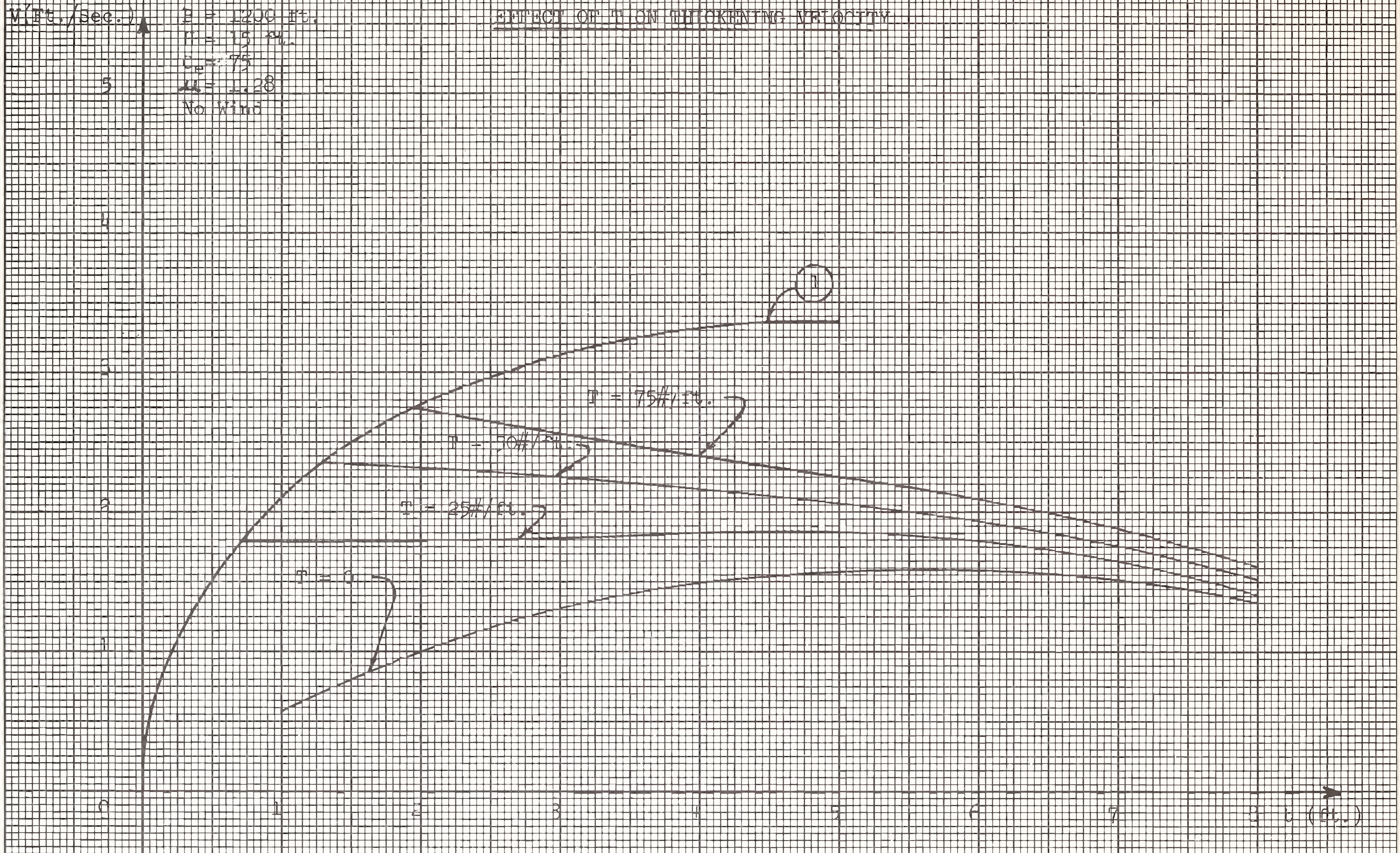
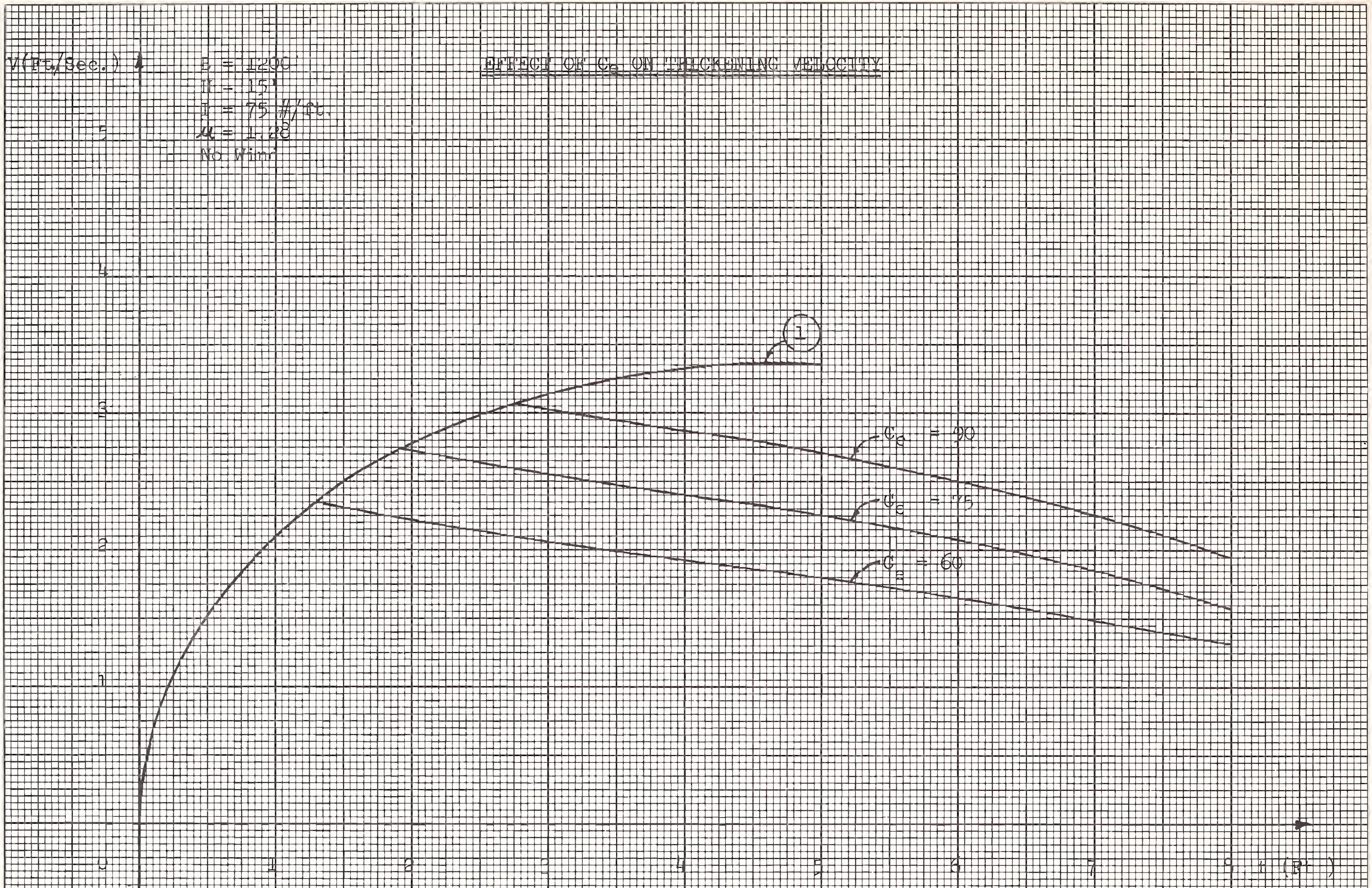


FIGURE 6-8

for the same river section equation 6-25 was plotted varying the bank cohesion from 0 to 75 #/ft. in 25#/ft. intervals. From figure 6-8 it can be seen that, for any value of cohesion assumed greater than 25#/ft. for this channel section, an unstable condition would result if the velocity is such that thickening by shove is required for equilibrium. A value of bank cohesion assumed below 25#/ft. would result in a more stable situation whereby some thickening by shove could occur without jamming. However, the jamming velocity is reduced considerably as the cohesion is also reduced.

It was pointed out by Morton[27] that the width of transition from a narrow to a wide cover was also quite sensitive to the value assumed for the effective roughness of the ice cover. This is shown on figure 6-9 where equation 6-25 was plotted for the same cross-section neglecting the wind and using 3 different values for the roughness coefficient, 60, 75 and 90. Because of the sensitivity of the transition from a narrow to a wide cover resulting from changes in cover roughness, Morton proposes to use the energy gradient under the cover as the criteria for the determination of the stability of the ice cover.

The determination of the type and final thickness of ice cover is shown to be dependent to a greater extent on the transitional width which in turn is shown by figures 6-8 and 6-9 to be quite sensitive to changes in the constants of equation 6-25. Thus it is



EFFECT OF C_e ON THICKENING VELOCITY

FIGURE 6-9

further demonstrated that there is a need for more data on both the ice cover roughness as well as the value for bank cohesion and friction along with a knowledge of any increase in internal resistance to shove before accurate calculations can be expected using the theory so developed.

Referring back to figure 6-7 curves 2 and 3, it can also be seen that the wind force may be significant. For this case, the velocity of flow at which shoving occurs is reduced from 2.75 ft./sec. with no wind to 2.4 ft./sec. when an average wind is included, a reduction of around 13%. Thus, a further study should include a study of the effects of wind in an attempt to ascertain its true significance.

Conclusions:

The work presented in this report on a theory for the prediction of ice cover formations can be considered to be just the beginning of the work that will be necessary to develop a general and reliable method. When using the values of bank cohesion and friction obtained from the observations of the ice cover formation on the St. Lawrence River and the Beauharnois Canal, the present theory results in an empirical relationship which is felt only to be valid for those waters.

Three types of ice cover formation from drift ice were discussed. The first was the normal progression as a narrow ice

cover or the normal thickening of an ice cover by shove with no jamming or appreciable decrease in flow. The second was the hanging-dam which is the thickening of the cover by shove causing jamming, a rise in stage and a decrease in the velocity of flow. The third is the overflow gorge which is the result of thickening of the cover by shove causing jamming, and overflow of the river banks resulting in the flooding and the icing over of large areas of the river valley.

A bridge can be the cause of jamming together of the drift ice in the forming of the ice bridge necessary to start an ice cover build-up. This can occur even in locations which previously were free of ice covers during the entire ice season. The reason being that the bridge piers reduce the distance which the ice must bridge in forming an ice cover. Also, if a flooding gorge or hanging dam is a possibility considerable build-up of ice above the normal water level is possible requiring the bridge to be placed higher above the water. Thus, the ability to be able to predict the type and extent of an ice cover can be quite important in choosing of the location, pier spacing and height of a new bridge.

A lot of work in gathering field data and modifying the present theory is yet necessary. This work includes the analysis of the water velocity and level, meterological conditions, channel roughness, ice roughness, and thickness of ice cover all during

the time of its formation. A careful study must also be made of the values for bank cohesion and friction along with the increase in internal strength of the cover to resist shove resulting from inter-freezing of drift ice or development of an angle of internal friction. This must be done for many different rivers and under all kinds of conditions. Some work can perhaps be done experimentally in the laboratory, thus eliminating many of the difficulties encountered in field investigations. Verification of the basic fluid mechanics theory must also be done to be sure that it is applicable to this particular problem.

Additional study and, it is imagined, considerable modification or extension of the proposed theory will be necessary before a complete understanding and accurate prediction can be made for rivers which are likely to form hanging dams or flooding gorges. For both of these types of covers, the raising of the water level will carry along with it the ice cover and the increased height of the ice could cause considerable trouble if it comes into contact with the bridge deck.

Even after a general theory is developed, many reaches of the river would still be much too complex for it to be applicable. For these areas it would be necessary to physically study the individual sites or possibly make controlled model studies.

SUMMARY

The types of ice and ice covers that may form on the river and streams in the northern latitudes of the United States have been described. The types of ice include frazil, anchor, and sheet ice. The types of ice covers included smooth sheet ice on still or slow moving water and covers made up of moving drift ice on streams with higher velocities of flow.

A method to predict the formation of an ice cover from drift ice was presented and discussed. Only normal thickening of the cover or thickening by shove with no jamming was covered by this theory. Jamming which could cause hanging dams or ice gorges are not fully explained with the present theory.

Also, with the limited information available, an attempt was made to describe the manner in which this ice causes forces on the bridges.

Along with this discussion, a development of some of the theory for the predicting of maximum forces was given. Values for the loads which could possibly occur from ice for hypothetical bridges and bridge sites were calculated for the three sources of loading which include jamming, expansion, and moving of the ice.

The need for a great deal of additional work in all areas of this field before a complete understanding of these phenomena is

acquired was emphasized throughout the report. Much of the work is of a very basic nature, such as the obtaining of reliable mechanical properties for the river ice. No attempt will be made here to tabulate these many areas which have been mentioned previously in the report where additional work is felt needed. The two broad areas which the authors feel would be most beneficial in obtaining a better understanding of the problems here in Montana are the areas of developing criteria for the prediction of ice cover formation and the behavior of moving ice as it comes into contact with the bridge piers.

It is not possible at this time, with the present available information, to develop rational design criteria for determining of forces of ice on bridges. However, there are certain things which should be considered in the choosing of a bridge site and designing the bridge.

First, a careful winter study should be made in an area where a bridge is desired, checking various alternate crossings. Old time residents of the area and local highway personnel should be interviewed to determine where and how the ice generally forms, where and how ice jamming occurs, maximum height to which the ice has been known to form or jam, and the extent of flooding that has occurred in the area during the ice season. This information is felt to be very useful. Then, if possible, a site should be chosen where there is the least possibility of ice forming or

jamming since it is not felt that the bridge itself is going to contribute much to the problem as was mentioned previously unless the bridge abutments are moved into the channel causing a considerable constriction. A knowledge of the height of ice build-up and jamming is important since most problems and damage appear to occur when the ice builds up and impinges on the deck structure.

If there is a possibility of sheet ice forming between the piers, expansive force from the ice is also a possibility. It may, however, be possible to eliminate this problem if the channel is altered so that the velocity of flow is higher than that which will permit a smooth cover to develop.

In areas where ice may be a problem, the ice breaking bridge pier should be used and care should be exercised to place it parallel with the flow of the river if at all possible. The height of the bridge should be placed well above the height of the maximum expected height of ice jam or build-up. The lateral force to use in the design of the pier can at best be little more than a guess. However, from the study of the site it is possible to determine from what source the load may occur. If the loading comes from the expansion of ice, an estimate can be made knowing the characteristics of the particular site and bridge. This load could be as much as 20 k/ft. of width of pier. Loads from moving ice cakes can be estimated by knowing the approximate maximum size floes and flow velocity and characteristics. Loads from jams can be

estimated using the theory presented in chapter 3 using the limiting depth of jam as described in that chapter.

Most, if not all of these factors, it appears, are being taken into consideration at the present time by the Montana State Highway Department. However, it is hoped that the discussion on the lateral loads on bridge piers will be helpful in their estimating of loads for design purposes. It is also hoped that this report will stimulate additional work in this field and that eventually rational design criteria can be developed.

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